

Polynomials

How could you solve this problem?

Denali and Mahala weed the borders on the north and south sides of their rectangular yard. Denali starts first and has weeded 1 m on the south side when Mahala says he should be weeding the north side. So, Denali moves to the north side. Mahala finishes weeding the south side. Then she moves to the north side where she weeds 2 m. Both students have then finished. Which student weeded more of the borders? How much more?



What You'll Learn

- Recognize, write, describe, and classify polynomials.
- Use algebra tiles, pictures, and algebraic expressions to represent polynomials.
- Strategies to add and subtract polynomials.
- Strategies to multiply and divide a polynomial by a monomial.

Why It's Important

Just as numbers are the building blocks of arithmetic, polynomials are the building blocks of algebra. In later grades, you will use polynomials to model real-world situations in business, science, medicine, and engineering. The skills, understanding, and language that you develop in this unit will lay the foundation for this work.



Key Words

- polynomial
- term
- coefficient
- degree
- constant term
- monomial
- binomial
- trinomial
- like terms

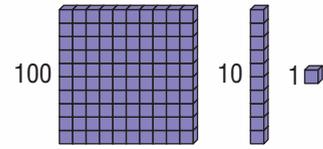
5.1

Modelling Polynomials

FOCUS

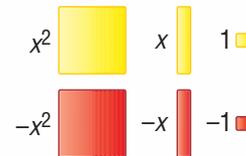
- Model, write, and classify polynomials.

In arithmetic, we use Base Ten Blocks to model whole numbers. How would you model the number 234?



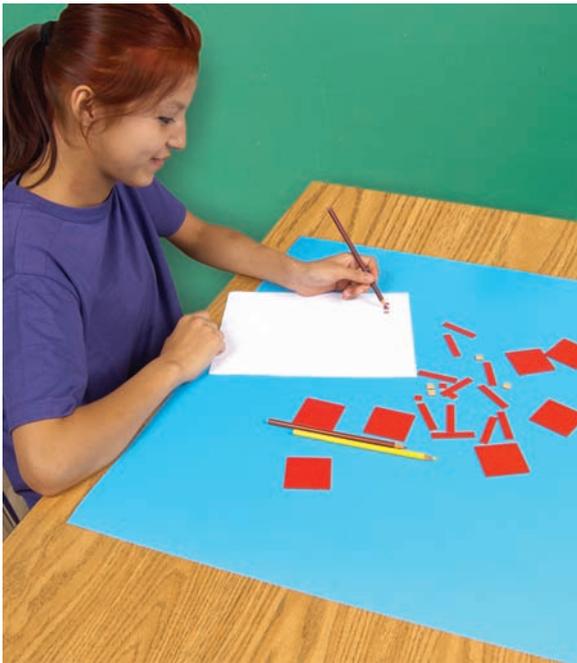
In algebra, we use algebra tiles to model integers and variables.

Yellow represents positive tiles. Red represents negative tiles.



How are Base Ten Blocks and algebra tiles alike?

Investigate



Use algebra tiles.

- Model each expression. Sketch the tiles. How do you know which tiles to use? How do you know how many of each tile to use?

- $x^2 + x - 3$
- $-2x^2 - 3$
- $2x^2 + 3x$
- $-2x^2 - 3x + 1$
- $-3x + 3$

- Write your own expression. Have your partner model it with tiles. Model your partner's expression with tiles.

Reflect & Share

For the first activity, compare your sketches with those of another pair of students.

Did you use the same tiles each time? If not, is one of you wrong? Could both of you be correct? Explain.

Did the order in which you laid out the tiles matter? Explain.

Connect

We can use algebra tiles to model an expression such as $3x^2 - 2x + 5$.

To model $3x^2 - 2x + 5$, we use three x^2 -tiles, two $-x$ -tiles, and five 1-tiles.



A **polynomial** is one term or the sum of terms whose variables have whole-number exponents.

The expression $3x^2 - 2x + 5 = 3x^2 + (-2)x + 5$ is an example of a polynomial in the variable x . This polynomial has 3 terms: $3x^2$, $(-2)x$, and 5

Terms are numbers, variables, or the product of numbers and variables.

The **coefficients** of the variable are 3 and -2 .

The term with the greatest exponent determines the **degree** of the polynomial. This polynomial has degree 2.

$$3x^2 - 2x + 5$$

The term 5 is a **constant term**. Its value does not change when the value of x changes. A constant term has degree 0.

The term $-2x$ has degree 1 because $-2x = -2x^1$.

We can use any variable to write a polynomial and to describe the tiles that model it.

For example, the tiles that model the polynomial $-5n^2 + 7n - 1$ also model the polynomial $-5p^2 + 7p - 1$.

We can also classify a polynomial by the number of terms it has.

Polynomials with 1, 2, or 3 terms have special names.

A **monomial** has 1 term; for example: $4a$, 6 , $-2p^2$

A **binomial** has 2 terms; for example: $2c - 5$, $2m^2 + 3m$

A **trinomial** has 3 terms; for example: $2h^2 - 6h + 4$

A polynomial is usually written in descending order; that is, the exponents of the variable decrease from left to right;

for example, the polynomial $2k - 4k^2 + 7$ is written as $-4k^2 + 2k + 7$.

An algebraic expression that contains a term with a variable in the denominator, such as $\frac{3}{n}$, or the square root of a variable, such as \sqrt{n} , is *not* a polynomial.

Example 1 Recognizing the Same Polynomials in Different Variables

Which of these polynomials can be represented by the same algebra tiles?

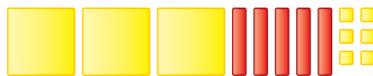
- a) $3x^2 - 5x + 6$ b) $-5 + 6r + 3r^2$ c) $-5m + 6 + 3m^2$

Justify the answer.

A Solution

- a) $3x^2 - 5x + 6$

Use three x^2 -tiles, five $-x$ -tiles, and six 1-tiles.



- b) $-5 + 6r + 3r^2$

Use five -1 -tiles, six r -tiles, and three r^2 -tiles.



- c) $-5m + 6 + 3m^2$

Use five $-m$ -tiles, six 1-tiles, and three m^2 -tiles.



In parts a and c, the same algebra tiles are used.

So, the polynomials $3x^2 - 5x + 6$ and $-5m + 6 + 3m^2$ can be represented by the same tiles.

Example 2 Modelling Polynomials with Algebra Tiles

Use algebra tiles to model each polynomial.

Is the polynomial a monomial, binomial, or trinomial? Explain.

- a) $-2x^2$ b) $2b^2 - b + 4$ c) $5a - 3$

A Solution

- a) To represent $-2x^2$, use two $-x^2$ -tiles.

Since there is only one type of tile, $-2x^2$ is a monomial.



- b) To represent $2b^2 - b + 4$, use two b^2 -tiles, one $-b$ -tile, and four 1-tiles.

Since there are 3 types of tiles, $2b^2 - b + 4$ is a trinomial.



- c) To represent $5a - 3$, use five a -tiles and three -1 -tiles. Since there are 2 types of tiles, $5a - 3$ is a binomial.

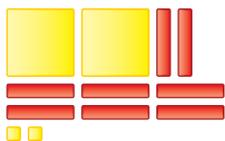


Two polynomials are *equivalent* when they can be represented by identical algebra tiles.

Example 3 Recognizing Equivalent Polynomials

a) Which polynomial does each group of algebra tiles represent?

Model A



Model B



Model C



b) Which of the polynomials in part a are equivalent? How do you know?

A Solution

a) Use a table.

Model	Description of Tiles	Polynomial
A	two x^2 -tiles, eight $-x$ -tiles, and two 1-tiles	$2x^2 - 8x + 2$
B	eight $-x$ -tiles, two x^2 -tiles, and two 1-tiles	$-8x + 2x^2 + 2$
C	four $-x$ -tiles and six 1-tiles	$-4x + 6$

b) Both models A and B contain the same tiles. The polynomials represented by these tiles have the same degree, and the same terms: $2x^2$, $-8x$, and 2. Both polynomials can be written as: $2x^2 - 8x + 2$. So, $2x^2 - 8x + 2$ and $-8x + 2x^2 + 2$ are equivalent polynomials. Model C has no x^2 -tiles, so its degree is different from that of models A and B.

Discuss the ideas

1. In the polynomial $3 + 2p$, which term is the constant term? How are constant terms modelled with algebra tiles?
2. Suppose you are given an algebra tile model of a polynomial. How can you identify the terms, the coefficients, and the degree of the polynomial? How can you identify the constant term?
3. What do we mean by “equivalent polynomials”? How can you determine whether two polynomials are equivalent?

Practice

Check

4. Which of the following expressions are polynomials? Explain how you know.
- a) $2 + 3n$ b) $3\sqrt{x}$
 c) $-5m + 1 + 2m^2$ d) 7
 e) $\frac{1}{x^2} + \frac{1}{x} + 1$ f) $\frac{1}{2}s$
5. Is each expression a monomial, binomial, or trinomial? Explain how you know.
- a) $3t + 4t^2 - 2$ b) $5 - 3g$
 c) $9k$ d) 11
6. Name the coefficient, variable, and degree of each monomial.
- a) $-7x$ b) $14a^2$
 c) m d) 12
7. Identify the degree of each polynomial. Justify your answers.
- a) $7j^2 + 4$ b) $9x$
 c) $2 - 5p + p^2$ d) -10

Apply

8. Identify the polynomials that can be represented by the same set of algebra tiles.
- a) $x^2 + 3x - 4$
 b) $-3 + 4n - n^2$
 c) $4m - 3 + m^2$
 d) $-4 + r^2 + 3r$
 e) $-3m^2 + 4m - 3$
 f) $-h^2 - 3 + 4h$
9. Name the coefficients, variable, and degree of each polynomial. Identify the constant term if there is one.
- a) $5x^2 - 6x + 2$ b) $7b - 8$
 c) $12c^2 + 2$ d) $12m$
 e) 18 f) $3 + 5x^2 - 8x$

10. One student says, “ $4a$ is a monomial.” Another student says, “ $4a$ is a polynomial.” Who is correct? Explain.
11. Use algebra tiles to model each polynomial. Sketch the tiles.
- a) $4x - 3$
 b) $-3n - 1$
 c) $2m^2 + m + 2$
 d) $-7y$
 e) $-d^2 - 4$
 f) 3
12. Match each polynomial with its corresponding algebra tile model.
- a) $r^2 - r + 3$
 b) $-t^2 - 3$
 c) $-2v$
 d) $2w + 2$
 e) $2s^2 - 2s + 1$

Model A



Model B



Model C



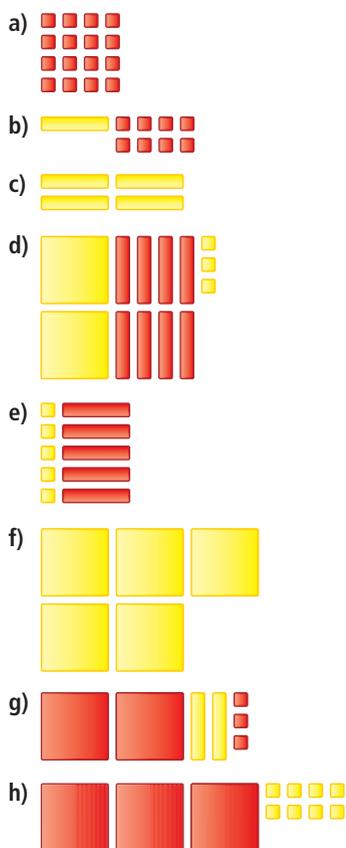
Model D



Model E

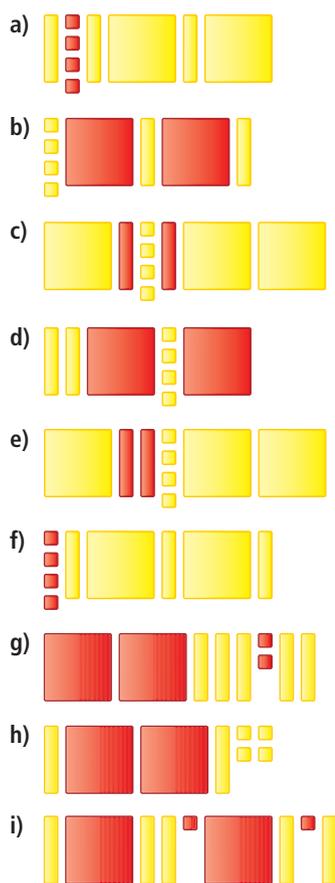


- 13.** Which polynomial does each collection of algebra tiles represent?
Is the polynomial a monomial, binomial, or trinomial? Explain.



- 14.** Write a polynomial with the given degree and number of terms. Use algebra tiles to model the polynomial. Sketch the tiles.
- degree 1, with 2 terms
 - degree 0, with 1 term
 - degree 2, with 1 term
 - degree 2, with 3 terms and constant term 5

- 15.** Identify which polynomials are equivalent.
Explain how you know.



- 16.** Identify which polynomials are equivalent.
Justify your answers.
- $5 - v + 7v^2$
 - $7v + 5 - v^2$
 - $5v + v^2 - 7$
 - $-7 + 5v + v^2$
 - $5 - v^2 + 7v$
 - $7v^2 + v + 5$

- 17.** Write an expression that is *not* a polynomial.
Explain why it is not a polynomial.

18. Assessment Focus

- a) Use algebra tiles to model each polynomial. Sketch the tiles. Identify the variable, degree, number of terms, and coefficients.
- $-2x - 3x^2 + 4$
 - $m^2 + m$
- b) Write a polynomial that matches this description:
a polynomial in variable c , degree 2, binomial, constant term -5
- c) Write another polynomial that is equivalent to the polynomial you wrote in part b. Explain how you know that the polynomials are equivalent.
19. a) Write as many polynomials as you can that are equivalent to $-8d^2 - 3d - 4$. How do you know you have written all possible polynomials?
- b) Which polynomial in part a is in descending order? Why is it useful to write a polynomial in this form?

Take It Further

20. The *stopping distance* of a car is the distance the car travels between the time the driver applies the brakes and the time the car stops. The polynomial $0.4s + 0.02s^2$ can be used to calculate the stopping distance in metres of a car travelling at s kilometres per hour on dry pavement.
- Determine the stopping distance for each speed:
i) 25 km/h ii) 50 km/h iii) 100 km/h
 - Does doubling the speed double the stopping distance? Explain.



Reflect

What is a polynomial?

How can you represent a polynomial with algebra tiles and with symbols?

Include examples in your explanation.

Math Link

Your World

A polynomial can be used to model projectile motion. When a golf ball is hit with a golf club, the distance the ball travels in metres, in terms of the time t seconds that it is in the air, may be modelled by the polynomial $-4.9t^2 + 22.8t$.



5.2

Like Terms and Unlike Terms

FOCUS

- Simplify polynomials by combining like terms.

When you work with integers,
a 1-tile and a -1 -tile form a zero pair. 

What do you think happens when you combine algebra tiles with opposite signs?
Which expression do these tiles represent?



Investigate



You will need algebra tiles and a paper bag.

- Put both colours of algebra tiles in a bag. Take a handful of tiles and sketch them. Construct a table to record your work.

Algebra Tile Model	Symbolic Record

Use symbols to write the polynomial modelled by the tiles. Remove zero pairs.

Sketch the tiles that remain.

Use symbols to write the polynomial represented by the smaller set of tiles.

- Return the algebra tiles to the bag. Repeat the activity 4 more times.



Reflect & Share

Share your results with another pair of students.

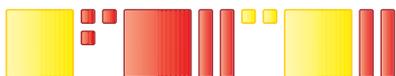
How could you verify each other's results?

When can you remove zero pairs from a set of tiles?

How does removing zero pairs help you simplify the polynomial that represents the set of tiles?

Connect

Here is a collection of red and yellow algebra tiles:

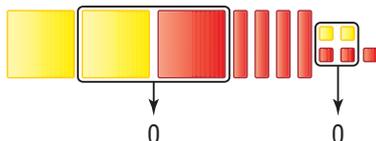


We organize the tiles by grouping like tiles:



These tiles represent the polynomial: $2x^2 - x^2 - 4x + 2 - 3$

We simplify the tile model by removing zero pairs.



The remaining tiles represent the polynomial: $x^2 - 4x - 1$

We say that the polynomial $2x^2 - x^2 - 4x + 2 - 3$ *simplifies* to $x^2 - 4x - 1$.

A polynomial is in *simplified form* when:

- its algebra tile model uses the fewest tiles possible
- its symbolic form contains only one term of each degree and no terms with a zero coefficient

Terms that can be represented by algebra tiles with the same size and shape are called **like terms**.

$-x^2$ and $3x^2$ are like terms.

Each term is modelled with x^2 -tiles.

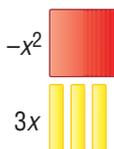
Each term has the same variable, x , raised to the same exponent, 2.



$-x^2$ and $3x$ are *unlike terms*.

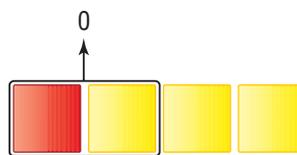
Each term is modelled with a different algebra tile.

Each term has the variable x , but the exponents are different.



To simplify a polynomial, we group like terms and remove zero pairs.

$$-x^2 + 3x^2 \text{ simplifies to } 2x^2.$$



We can also simplify a polynomial by adding the coefficients of like terms. This is called *combining like terms*.

$$\begin{aligned} -x^2 + 3x^2 &= -1x^2 + 3x^2 && \text{Add the integer coefficients: } -1 + 3 = 2 \\ &= 2x^2 \end{aligned}$$

The polynomials $-x^2 + 3x^2$ and $2x^2$ are *equivalent*.

So, a polynomial in simplified form is also the equivalent polynomial in which all the like terms have been combined.

$-x^2 + 3x$ cannot be simplified.

We may not add coefficients when we have unlike terms.



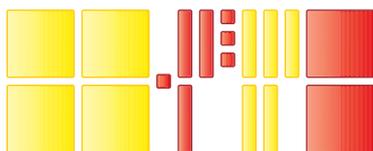
Example 1 Using Algebra Tiles to Simplify a Polynomial

Use algebra tiles to simplify the polynomial $4n^2 - 1 - 3n - 3 + 5n - 2n^2$. Record the process symbolically.

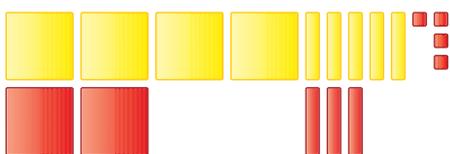
A Solution

Tile Model

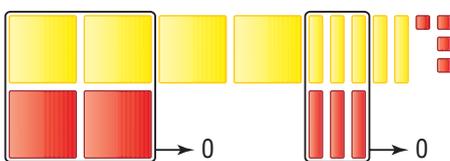
Display $4n^2 - 1 - 3n - 3 + 5n - 2n^2$.



Group like tiles.



Remove zero pairs.



The remaining tiles represent $2n^2 + 2n - 4$.

Symbolic Record

$$4n^2 - 1 - 3n - 3 + 5n - 2n^2$$

Group like terms:

$$4n^2 - 2n^2 + 5n - 3n - 1 - 3$$

Combine like terms:

$$2n^2 + 2n - 4$$

Example 2 Simplifying a Polynomial Symbolically

Simplify: $14x^2 - 11 + 30x + 3 + 15x - 25x^2$

A Solution

We need many tiles to model this polynomial.

So, we simplify it symbolically.

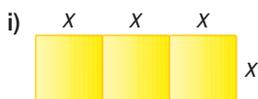
$$\begin{aligned}
 & 14x^2 - 11 + 30x + 3 + 15x - 25x^2 && \text{Group like terms.} \\
 = & 14x^2 - 25x^2 + 30x + 15x - 11 + 3 && \text{Add the coefficients of like terms.} \\
 = & -11x^2 + 45x - 8
 \end{aligned}$$

In *Example 2*, the polynomials $14x^2 - 11 + 30x + 3 + 15x - 25x^2$ and $-11x^2 + 45x - 8$ are equivalent.

Polynomials can be used to represent measures such as the side lengths of shapes.

Example 3 Investigating Situations that Represent Polynomials

a) Write a polynomial to represent the perimeter of each rectangle.



b) Each polynomial represents the perimeter of a rectangle.

Use algebra tiles to make the rectangle.

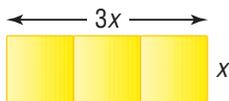
i) $4a + 2$

ii) $10b$

A Solution

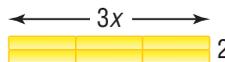
a) i) The dimensions of the rectangle are $3x$ and x . So, the perimeter of the rectangle is:

$$3x + x + 3x + x = 8x$$



ii) The dimensions of the rectangle are $3x$ and 2 . So, the perimeter of the rectangle is:

$$3x + 2 + 3x + 2 = 6x + 4$$



b) i) The perimeter is $4a + 2$.

Work backward.

Write the polynomial as the sum of equal pairs of terms.

$$4a + 2 = 2a + 2a + 1 + 1$$

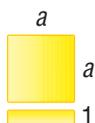
The dimensions of the rectangle could be $2a$ and 1 .



Another solution is:

$$4a + 2 = a + (a + 1) + a + (a + 1)$$

The dimensions of the rectangle could be a and $a + 1$.

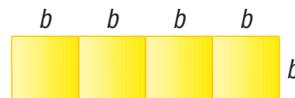


ii) The perimeter is $10b$.

Write the polynomial as the sum of equal pairs of terms.

$$10b = 4b + 4b + b + b$$

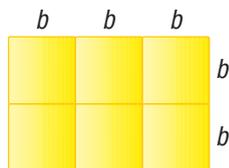
The dimensions of the rectangle could be $4b$ and b .



Another solution is:

$$10b = 3b + 3b + 2b + 2b$$

The dimensions of the rectangle could be $3b$ and $2b$.



A polynomial may contain more than one variable. Here is a polynomial in x and y :

$$-2x^2 + 3xy + y^2 - 4x - 8y$$

Example 4 Simplifying a Polynomial in Two Variables

Simplify: $4xy - y^2 - 3x^2 + 2xy - x - 3y^2$

A Solution

$$\begin{aligned} & 4xy - y^2 - 3x^2 + 2xy - x - 3y^2 && \text{Group like terms.} \\ = & 4xy + 2xy - y^2 - 3y^2 - 3x^2 - x && \text{Combine like terms.} \\ = & 6xy - 4y^2 - 3x^2 - x \end{aligned}$$

Discuss the ideas

1. Why can we combine like terms? Why can we not combine unlike terms?
2. How can you identify and combine like terms in an algebra tile model?
3. How can you identify and combine like terms symbolically?

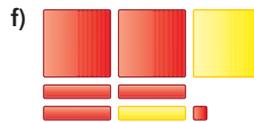
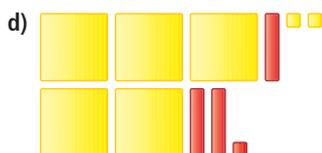
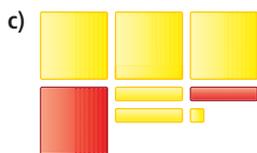
Practice

Check

4. a) Use algebra tiles to model $3d$ and $-5d$. Sketch the tiles.
 b) Are $3d$ and $-5d$ like terms? How can you tell from the tiles? How can you tell from the monomials?
5. a) Use algebra tiles to model $4p$ and $2p^2$. Sketch the tiles.
 b) Are $4p$ and $2p^2$ like terms? How can you tell from the tiles? How can you tell from the monomials?

Apply

6. From the list, which terms are like $8x$?
 $-3x$, $5x^2$, 4 , $3x$, 9 , $-11x^2$, $7x$, -3
 Explain how you know they are like terms.
7. From the list, which terms are like $-2n^2$?
 $3n$, $-n^2$, -2 , $4n$, $2n^2$, -2 , 3 , $5n^2$
 Explain how you know they are like terms.
8. For each part, combine tiles that represent like terms.
 Write the simplified polynomial.



9. Identify the equivalent polynomials in the diagrams below. Justify your answers.



10. A student made these mistakes on a test.

► The student simplified

$$2x + 3x \text{ as } 5x^2.$$

► The student simplified

$$4 + 3x \text{ as } 7x.$$

Use algebra tiles to explain what the student did wrong.

What are the correct answers?

11. Use algebra tiles to model each polynomial, then combine like terms. Sketch the tiles.

- $2c + 3 + 3c + 1$
- $2x^2 + 3x - 5x$
- $3f^2 + 3 - 6f^2 - 2$
- $3b^2 - 2b + 5b + 4b^2 + 1$
- $5t - 4 - 2t^2 + 3 + 6t^2$
- $4a - a^2 + 3a - 4 + 2a^2$

12. Simplify each polynomial.

- $2m + 4 - 3m - 8$
- $4 - 5x + 6x - 2$
- $3g - 6 - 2g + 9$
- $-5 + 1 + h - 4h$
- $-6n - 5n - 4 - 7$
- $3s - 4s - 5 - 6$

13. Simplify each polynomial.

- $6 - 3x + x^2 + 9 - x$
- $5m - 2m^2 - m^2 + 5m$
- $5x - x^2 + 3x + x^2 - 7$
- $3p^2 - 2p + 4 + p^2 + 3$
- $a^2 - 2a - 4 + 2a - a^2 + 4$
- $-6x^2 + 17x - 4 - 3x^2 + 8 - 12x$

14. Simplify each polynomial.

- $3x^2 + 5y - 2x^2 - 1 - y$
- $pq - 1 - p^2 + 5p - 5pq - 2p$
- $5x^2 + 3xy - 2y - x^2 - 7x + 4xy$
- $3r^2 - rs + 5s + r^2 - 2rs - 4s$
- $4gh + 7 - 2g^2 - 3gh - 11 + 6g$
- $-5s + st - 4s^2 - 12st + 10s - 2s^2$

15. Identify the equivalent polynomials.

Justify your answers.

- $1 + 5x$
- $6 - 2x + x^2 - 1 - x + x^2$
- $4x^2 - 7x + 1 - 7x^2 + 2x + 3$
- $4 - 5x - 3x^2$
- $2x^2 - 3x + 5$
- $3x + 2x^2 + 1 - 2x^2 + 2x$

16. Write 3 different polynomials that simplify to $-2a^2 + 4a - 8$.

17. Write a polynomial with degree 2 and 5 terms, which has only 2 terms when it is simplified.

18. Assessment Focus

- A student is not sure whether $x + x$ simplifies to $2x$ or x^2 . Explain how the student can use algebra tiles to determine the correct answer. What is the correct answer?
- Simplify each polynomial. How do you know that your answers are correct?
 - $-2 + 4r - 2r + 3$
 - $2t^2 - 3t + 4t^2 - 6t$
 - $3c^2 + 4c + 2 + c^2 + 2c + 1$
 - $15x^2 - 12xy + 5y + 10xy - 8y - 9x^2$
- Create a polynomial that cannot be simplified. Explain why it cannot be simplified.

19. Write a polynomial to represent the perimeter of each rectangle.

-
-
-
-

20. Each polynomial below represents the perimeter of a rectangle. Use algebra tiles to make the rectangle. Sketch the tiles. How many different rectangles can you make each time?

- a) $6c + 4$ b) $4d$ c) $8 + 2m$
 d) $12r$ e) $6s$ f) $4a + 10$

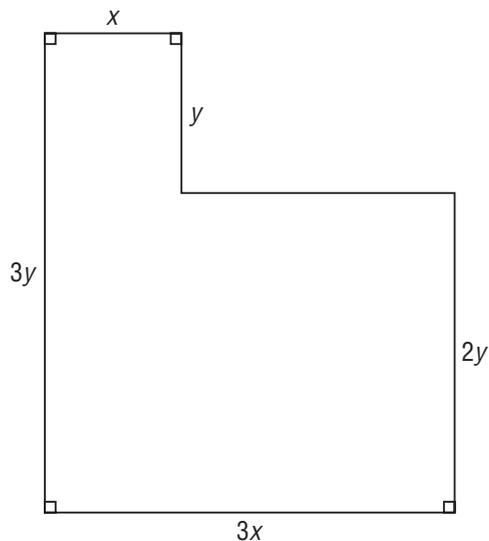
Take It Further

21. Many algebra tile kits contain x -tiles and y -tiles.

-  x
 $-x$
 y
 $-y$

What do you think an xy -tile looks like? Sketch your idea and justify your picture.

22. Write a polynomial for the perimeter of this shape. Simplify the polynomial.



Reflect

Explain how like terms can be used to simplify a polynomial. Use diagrams and examples in your explanation.

Math Link

Your World

On a forward somersault dive, a diver's height above the water, in metres, in terms of the time t seconds after the diver leaves the board may be modelled by the polynomial $-4.9t^2 + 6t + 3$.



5.3

Adding Polynomials

FOCUS

- Use different strategies to add polynomials.



Investigate

2

You will need algebra tiles and a paper bag.
Conduct the activity 3 times.

Put both colours of algebra tiles in a paper bag.
Each person removes a handful of algebra tiles and
writes the simplified polynomial that the tiles model.
Add the two polynomials.
Record your work as an addition sentence.



Reflect & Share

Compare your strategies for adding two polynomials with those of another pair of students.
If you used different strategies, explain your strategies.
If you used the same strategies, find a pair of students who used a different strategy.
Which terms can be combined when you add polynomials?
Why can these terms be combined?

Connect

To add polynomials, we combine the algebra tiles that represent each polynomial and record the process symbolically. This develops a strategy to add polynomials without algebra tiles.

When we write the sum of two polynomials, we write each polynomial in brackets.

To determine the sum of $3x^2 + 2x + 4$ and $-5x^2 + 3x - 5$, we write:

$$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$$

Tile Model

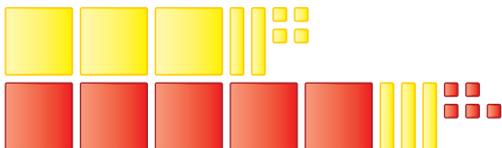
Display: $3x^2 + 2x + 4$



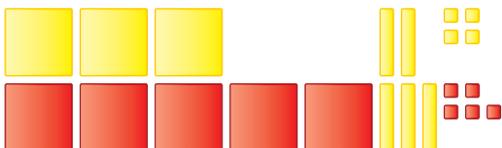
Display: $-5x^2 + 3x - 5$



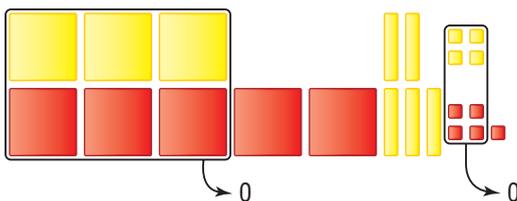
Combine the displays.



Group like tiles.



Remove zero pairs.



The remaining tiles represent

$$-2x^2 + 5x - 1.$$

Symbolic Record

The sum is:

$$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$$

This is written as:

$$3x^2 + 2x + 4 - 5x^2 + 3x - 5$$

Group like terms:

$$3x^2 - 5x^2 + 2x + 3x + 4 - 5$$

Combine like terms:

$$-2x^2 + 5x - 1$$

Example 1 Adding Polynomials Symbolically

Add: $(7s + 14) + (-6s^2 + s - 6)$

Solutions

Add the polynomials by adding the coefficients of like terms. In the second polynomial, the term s has coefficient 1, so write s as $1s$.

Method 1

Add horizontally.

$$\begin{aligned} &(7s + 14) + (-6s^2 + 1s - 6) && \text{Remove the brackets.} \\ &= 7s + 14 - 6s^2 + 1s - 6 && \text{Group like terms.} \\ &= -6s^2 + 7s + 1s + 14 - 6 && \text{Combine like terms by adding their coefficients.} \\ &= -6s^2 + 8s + 8 \end{aligned}$$

Method 2

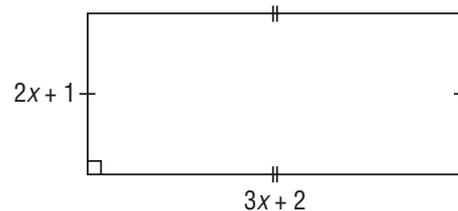
Add vertically. Align like terms, then add their coefficients.

$$\begin{array}{r} 7s + 14 \\ + -6s^2 + 1s - 6 \\ \hline -6s^2 + 8s + 8 \end{array}$$

So, $(7s + 14) + (-6s^2 + 1s - 6) = -6s^2 + 8s + 8$

Example 2 Determining a Polynomial for the Perimeter of a Rectangle

- a) Write a polynomial for the perimeter of this rectangle.
Simplify the polynomial.



- b) Substitute to check the answer.

A Solution

- a) The perimeter is the sum of the measures of the four sides.

$$\begin{array}{r} 2x + 1 \\ + 2x + 1 \\ + 3x + 2 \\ + 3x + 2 \\ \hline 10x + 6 \end{array}$$

The perimeter is $10x + 6$.

b) Choose a value for x , such as $x = 1$.

Write the addition sentence:

$$2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 = 10x + 6$$

Substitute $x = 1$.

Left side:

$$\begin{aligned} & 2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 \\ &= 2(1) + 1 + 2(1) + 1 + 3(1) + 2 + 3(1) + 2 \\ &= 2 + 1 + 2 + 1 + 3 + 2 + 3 + 2 \\ &= 16 \end{aligned}$$

Right side:

$$\begin{aligned} 10x + 6 &= 10(1) + 6 \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

Since the left side equals the right side, the polynomial for the perimeter is correct.

Example 3 Adding Polynomials in Two Variables

Add: $(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$

A Solution

$$\begin{aligned} & (2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2) \\ &= 2a^2 + a - 3b - 7ab + 3b^2 - 4b^2 + 3ab + 6b - 5a + 5a^2 \\ &= 2a^2 + 5a^2 + a - 5a - 3b + 6b - 7ab + 3ab + 3b^2 - 4b^2 \\ &= 7a^2 - 4a + 3b - 4ab - b^2 \end{aligned}$$

Remove brackets.
Group like terms.
Combine like terms.

Discuss the ideas

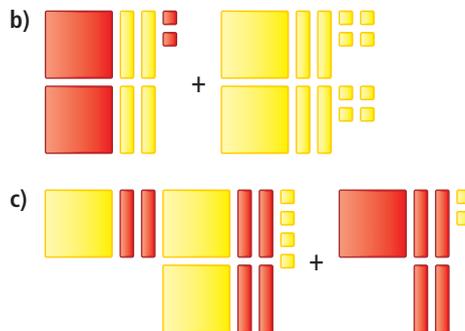
1. How can you use what you know about adding integers to add polynomials?
2. How is adding polynomials like simplifying a polynomial?

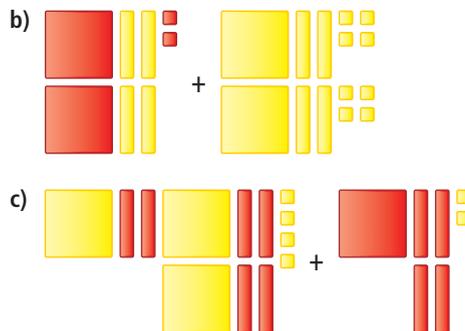
Practice

Check

3. Write the polynomial sum modelled by each set of tiles.

a) 

b) 

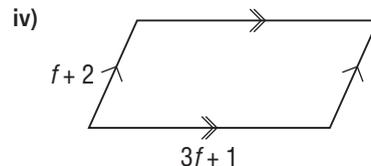
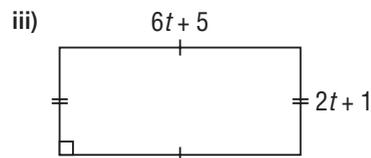
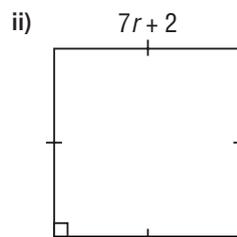
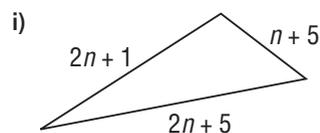
c) 

4. Explain how to use algebra tiles to determine $(3x^2 + 2) + (x^2 - 1)$. What is the sum?
5. Use algebra tiles to model each sum of binomials. Record your answer symbolically.
- $(5g + 3) + (2g + 4)$
 - $(3 - 2j) + (-4 + 2j)$
 - $(p + 1) + (5p - 6)$
 - $(7 + 4m) + (-5m + 4)$
6. Add these polynomials. Visualize algebra tiles if it helps.
- $$\begin{array}{r} 2x + 4 \\ + 3x - 5 \\ \hline \end{array}$$
 - $$\begin{array}{r} 3x^2 + 5x \\ + -2x^2 - 8x \\ \hline \end{array}$$
 - $$\begin{array}{r} 3x^2 + 5x + 7 \\ + -8x^2 - 3x + 5 \\ \hline \end{array}$$
7. Do you prefer to add vertically or horizontally? Give reasons for your choice.

Apply

8. Use a personal strategy to add.
- $(6x + 3) + (3x + 4)$
 - $(5b - 4) + (2b + 9)$
 - $(6 - 3y) + (-3 - 2y)$
 - $(-n + 7) + (3n - 2)$
 - $(-4s - 5) + (6 - 3s)$
 - $(1 - 7h) + (-7h - 1)$
 - $(8m + 4) + (-9 + 3m)$
 - $(-8m - 4) + (9 - 3m)$
9. Add. Which strategy did you use each time?
- $(4m^2 + 4m - 5) + (2m^2 - 2m + 1)$
 - $(3k^2 - 3k + 2) + (-3k^2 - 3k + 2)$
 - $(-7p - 3) + (p^2 + 5)$
 - $(9 - 3t) + (9t + 3t^2 - 6t)$
 - $(3x^2 - 2x + 3) + (2x^2 + 4)$
 - $(3x^2 - 7x + 5) + (6x - 6x^2 + 8)$
 - $(6 - 7x + x^2) + (6x - 6x^2 + 10)$
 - $(1 - 3r + r^2) + (4r + 5 - 3r^2)$

10. a) For each shape below, write the perimeter:
- as a sum of polynomials
 - in simplest form



- b) Use substitution to check each answer in part a.

11. Sketch 2 different shapes whose perimeter could be represented by each polynomial.
- $8 + 6r$
 - $3s + 9$
 - $4 + 12t$
 - $20u$
 - $7 + 5v$
 - $4y + 6$
 - $9 + 9c$
 - $15m$

12. A student added $(4x^2 - 7x + 3)$ and $(-x^2 - 5x + 9)$ as follows.

Handwritten work on lined paper:

$$\begin{aligned} &(4x^2 - 7x + 3) + (-x^2 - 5x + 9) \\ &= 4x^2 - 7x + 3 - x^2 - 5x + 9 \\ &= 4x^2 - x^2 - 7x - 5x + 3 + 9 \\ &= 3x^2 - 2x + 1 \end{aligned}$$

Is the student's work correct?

If not, explain where the student made any errors and write the correct answer.

13. Assessment Focus

These tiles represent the sum of two polynomials.



- a) What might the two polynomials be?
Explain how you found out.
- b) How many different pairs of polynomials can you find? List all the pairs you found.
14. The sum of two polynomials is $12m^2 + 2m + 4$.
One polynomial is $4m^2 - 6m + 8$.
What is the other polynomial?
Explain how you found your answer.

Reflect

What strategies can you use for adding polynomials?
Which strategy do you prefer?
How can you check that your answers are correct?
Include examples in your explanation.

15. Create a polynomial that is added to $3x^2 + 7x + 2$ to get each sum.

- a) $5x^2 + 10x + 1$ b) $2x^2 + 5x + 8$
c) $4x^2 + 3x$ d) $-x^2 + x - 1$
e) $2x + 3$ f) 4

16. a) What polynomial must be added to $5x^2 + 3x - 1$ to obtain a sum of 0? Justify your answer.
b) How are the coefficients of the two polynomials related?
Will this relationship be true for all polynomials with a sum of 0? Explain.

17. Add.

- a) $(3x^2 - 2y^2 + xy) + (-2xy - 2y^2 - 3x^2)$
b) $(-5q^2 + 3p - 2q + p^2) + (4p + q + pq)$
c) $(3mn + m^2 - 3n^2 + 5m) + (7n^2 - 8n + 10)$
d) $(3 - 8f + 5g - f^2) + (2g^2 - 3f + 4g - 5)$

Take It Further

18. a) The polynomials $4x - 3y$ and $2x + y$ represent the lengths of two sides of a triangle. The perimeter of the triangle is $9x + 2$. Determine the length of the third side.
b) Use substitution to check your solution in part a.
19. The polynomial $5y + 3x + 7$ represents the perimeter of an isosceles triangle. Write three polynomials that could represent the side lengths of the triangle. Find as many answers as you can.

5.4

Subtracting Polynomials

FOCUS

- Use different strategies to subtract polynomials.

What strategies do you know to subtract two integers, such as $-2 - 3$?
How could these strategies help you subtract two polynomials?

Investigate



Use algebra tiles.

- Write two like monomials.
Subtract the monomials.
Write the subtraction sentence.
Subtract the monomials in the reverse order.
Write the new subtraction sentence.
Sketch the tiles you used.
- Repeat the process above for two binomials, then for two trinomials.
- Subtract. Use a strategy of your choice.
 $(5x) - (3x)$
 $(2x^2 + 3x) - (4x^2 - 6x)$
 $(3x^2 - 6x + 4) - (x^2 + 3x - 2)$
Use a different strategy to verify your answer.

Reflect & Share

Compare your answers and strategies with those of a pair of students who used a different strategy. Explain your strategies to each other. Work together to write an addition sentence that corresponds to each subtraction sentence.

Connect

Here are two strategies to subtract polynomials.

► Using algebra tiles

To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$

Use algebra tiles to model $3x^2 - 4x$.



To subtract $2x^2 - 6x$, we need to:

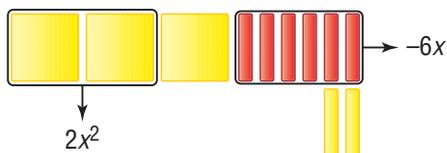
- Take away two x^2 -tiles from three x^2 -tiles.
- Take away six $-x$ -tiles from four $-x$ -tiles.

To do this, we need 2 more $-x$ -tiles.

So, we add 2 zero pairs of x -tiles.



Now we can take away the tiles for $2x^2 - 6x$.



The remaining tiles represent $x^2 + 2x$.

So, $(3x^2 - 4x) - (2x^2 - 6x) = x^2 + 2x$

► Using the properties of integers

We know that -6 is the opposite of 6 .

Subtracting -6 from an integer is the same as adding 6 to that integer.

The same process is true for like terms.

To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$

$$\begin{aligned} (3x^2 - 4x) - (2x^2 - 6x) &= 3x^2 - 4x - (2x^2) - (-6x) \\ &= 3x^2 - 4x - 2x^2 - (-6x) \\ &= 3x^2 - 4x - 2x^2 + 6x \\ &= 3x^2 - 2x^2 - 4x + 6x \\ &= x^2 + 2x \end{aligned}$$

Subtract each term.

Add the opposite term.

Collect like terms.

Combine like terms.

Example 1 Subtracting Two Trinomials

Subtract: $(-2a^2 + a - 1) - (a^2 - 3a + 2)$

Solutions

$$(-2a^2 + a - 1) - (a^2 - 3a + 2)$$

Method 1

Use algebra tiles.

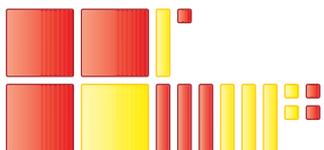
Display: $-2a^2 + a - 1$



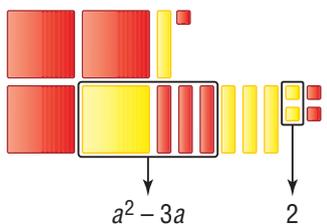
To subtract a^2 , add a zero pair of a^2 -tiles.

To subtract $-3a$, add 3 zero pairs of a -tiles.

To subtract 2, add 2 zero pairs of 1-tiles.



Now remove tiles for $a^2 - 3a + 2$.



The remaining tiles represent $-3a^2 + 4a - 3$.

Method 2

Use the properties of integers.

$$\begin{aligned} & (-2a^2 + a - 1) - (a^2 - 3a + 2) \\ &= -2a^2 + a - 1 - (a^2) - (-3a) - (+2) \\ &= -2a^2 + a - 1 - a^2 + 3a - 2 \\ &= -2a^2 - a^2 + a + 3a - 1 - 2 \\ &= -3a^2 + 4a - 3 \end{aligned}$$

To check the difference when two numbers are subtracted, we add the difference to the number that was subtracted; for example, to check that $23 - 5 = 18$ is correct, we add: $5 + 18 = 23$

We can use the same process to check the difference of two polynomials.

Example 2 Subtracting Trinomials in Two Variables

Subtract: $(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2)$

Check the answer.

▶ A Solution

$$\begin{aligned}(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2) &= 5x^2 - 3xy + 2y^2 - (8x^2) - (-7xy) - (-4y^2) \\ &= 5x^2 - 3xy + 2y^2 - 8x^2 + 7xy + 4y^2 \\ &= 5x^2 - 8x^2 - 3xy + 7xy + 2y^2 + 4y^2 \\ &= -3x^2 + 4xy + 6y^2\end{aligned}$$

To check, add the difference to the second polynomial:

$$\begin{aligned}(-3x^2 + 4xy + 6y^2) + (8x^2 - 7xy - 4y^2) &= -3x^2 + 4xy + 6y^2 + 8x^2 - 7xy - 4y^2 \\ &= -3x^2 + 8x^2 + 4xy - 7xy + 6y^2 - 4y^2 \\ &= 5x^2 - 3xy + 2y^2\end{aligned}$$

The sum is equal to the first polynomial.

So, the difference is correct.

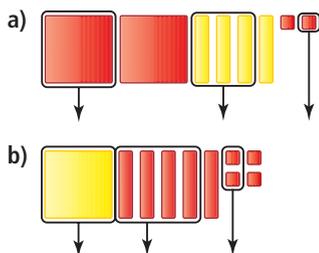
Discuss the ideas

1. How is subtracting polynomials like subtracting integers?
2. How is subtracting polynomials like adding polynomials? How is it different?
3. When might using algebra tiles not be the best method to subtract polynomials?

Practice

Check

4. Write the subtraction sentence that these algebra tiles represent.



5. Use algebra tiles to subtract.

Sketch the tiles you used.

- | | |
|-------------------|--------------------|
| a) $(5r) - (3r)$ | b) $(5r) - (-3r)$ |
| c) $(-5r) - (3r)$ | d) $(-5r) - (-3r)$ |
| e) $(3r) - (5r)$ | f) $(-3r) - (5r)$ |
| g) $(3r) - (-5r)$ | h) $(-3r) - (-5r)$ |

Apply

6. Use algebra tiles to model each difference of binomials. Record your answer symbolically.
- a) $(5x + 3) - (3x + 2)$
 - b) $(5x + 3) - (3x - 2)$
 - c) $(5x + 3) - (-3x + 2)$
 - d) $(5x + 3) - (-3x - 2)$

7. Use algebra tiles to model each difference of trinomials. Record your answer symbolically.

- $(3s^2 + 2s + 4) - (2s^2 + s + 1)$
- $(3s^2 - 2s + 4) - (2s^2 - s + 1)$
- $(3s^2 - 2s - 4) - (-2s^2 + s - 1)$
- $(-3s^2 + 2s - 4) - (2s^2 - s - 1)$

8. Use a personal strategy to subtract.

Check your answers by adding.

- $(3x + 7) - (-2x - 2)$
- $(b^2 + 4b) - (-3b^2 + 7b)$
- $(-3x + 5) - (4x + 3)$
- $(4 - 5p) - (-7p + 3)$
- $(6x^2 + 7x + 9) - (4x^2 + 3x + 1)$
- $(12m^2 - 4m + 7) - (8m^2 + 3m - 3)$
- $(-4x^2 - 3x - 11) - (x^2 - 4x - 15)$
- $(1 - 3r + r^2) - (4r + 5 - 3r^2)$

9. The polynomial $4n + 2500$ represents the cost, in dollars, to produce n copies of a magazine in colour. The polynomial $2n + 2100$ represents the cost, in dollars, to produce n copies of the magazine in black-and-white.

- Write a polynomial for the difference in the costs of the two types of magazines.
- Suppose the company wants to print 3000 magazines. How much more does it cost to produce the magazine in colour instead of black-and-white?

10. A student subtracted

$(2x^2 + 5x + 10) - (x^2 - 3)$ like this:

$$\begin{aligned} (2x^2 + 5x + 10) - (x^2 - 3) \\ = 2x^2 + 5x + 10 - x^2 + 3 \\ = x^2 + 8x + 10 \end{aligned}$$

- Use substitution to show that the answer is incorrect.
- Identify the errors and correct them.

11. **Assessment Focus** Create a polynomial subtraction question. Answer your question. Check your answer. Show your work.

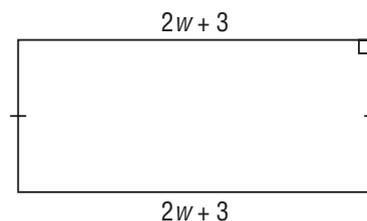
12. A student subtracted like this:

$$\begin{aligned} (2y^2 - 3y + 5) - (y^2 + 5y - 2) \\ = 2y^2 - 3y + 5 - y^2 + 5y - 2 \\ = 2y^2 - y^2 - 3y + 5y + 5 - 2 \\ = y^2 - 2y + 3 \end{aligned}$$

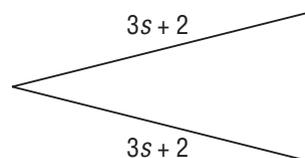
- Explain why the solution is incorrect.
- What is the correct answer? Show your work.
- How could you check that your answer is correct?
- What could the student do to avoid making the same mistakes in the future?

13. The perimeter of each polygon is given. Determine each unknown length.

a) $6w + 14$



b) $7s + 7$



c) $10p + 8$



14. a) Write two polynomials, then subtract them.
 b) Subtract the polynomials in part a in the reverse order.
 c) How do the answers in parts a and b compare? Why are the answers related this way?

15. Subtract.

- a) $(r^2 - 3rs + 5s^2) - (-2r^2 - 3rs - 5s^2)$
 b) $(-3m^2 + 4mn - n^2) - (5m^2 + 7mn + 2n^2)$
 c) $(5cd + 8c^2 - 7d^2) - (3d^2 + 6cd - 4c^2)$
 d) $(9e + 9f - 3e^2 + 4f^2) - (-f^2 - 2e^2 + 3f - 6e)$
 e) $(4jk - 7j - 2k + k^2) - (2j^2 + 3j - jk)$

16. The difference of two polynomials is

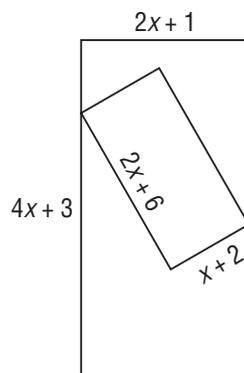
$$3x^2 + 4x - 7.$$

One polynomial is $-8x^2 + 5x - 4$.

- a) What is the other polynomial?
 b) Why are there two possible answers to part a?

Take It Further

17. The diagram shows one rectangle inside another rectangle. What is the difference in the perimeters of the rectangles?



18. One polynomial is subtracted from another. The difference is $-4x^2 + 2x - 5$. Write two polynomials that have this difference. How many different pairs of polynomials can you find? Explain.

Reflect

What strategy or strategies do you use to subtract polynomials?
 Why do you prefer this strategy or strategies?

Math Link

Your World

On a suspension bridge, the roadway is hung from huge cables passing through the tops of high towers. Here is a photograph of the Lions Gate Bridge in Vancouver. The position of any point on the cable can be described by its horizontal and vertical distance from the centre of the bridge. The vertical distance in metres is modelled by the polynomial $0.0006x^2$, where x is the horizontal distance in metres.



Mid-Unit Review

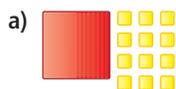
5.1

1. In each polynomial, identify: the variable, number of terms, coefficients, constant term, and degree.

- a) $3m - 5$
- b) $4r$
- c) $x^2 + 4x + 1$

2. Create a polynomial that meets these conditions: trinomial in variable m , degree 2, constant term is -5

3. Which polynomial is represented by each set of algebra tiles? Is the polynomial a monomial, binomial, or trinomial? How do you know?



4. Use algebra tiles to represent each polynomial. Sketch the tiles you used.

- a) $4n - 2$
- b) $-t^2 + 4t$
- c) $2d^2 + 3d + 2$

5.2

5. For each pair of monomials, which are like terms? Explain how you know.

- a) $2x, -5x$
- b) $3, 4g$
- c) $10, 2$
- d) $2q^2, -7q^2$
- e) $8x^2, 3x$
- f) $-5x, -5x^2$

6. Simplify $3x^2 - 7 + 3 - 5x^2 - 3x + 5$. Explain how you did this.

7. Renata simplified a polynomial and got $4x^2 + 2x - 7$. Her friend simplified the same polynomial and got $-7 + 4x^2 + 2x$. Renata thinks her friend's answer is wrong. Do you agree? Explain.

8. Cooper thinks that $5x - 2$ simplifies to $3x$. Is he correct? Explain. Use algebra tiles to support your explanation.

9. Identify the equivalent polynomials. Justify your answers.

- a) $1 + 3x - x^2$
- b) $1 + 3x^2 - x^2 + 2x - 2x^2 + x - 2$
- c) $x^2 - 3x - 1$
- d) $6 + 6x - 6x^2 - 4x - 5 + 2x^2 + x^2 - 4$
- e) $3x - 1$
- f) $-3x^2 + 2x - 3$
- g) $6x^2 - 6x - 6 + x - 5x^2 - 1 + 2x + 4$
- h) $3x - x^2 + 1$

5.3

5.4

10. Use algebra tiles to add or subtract. Sketch the tiles you used.

- a) $(4f^2 - 4f) + (-2f^2)$
- b) $(3r^2 + 2r + 5) + (-7r^2 + r - 3)$
- c) $(-2v + 5) - (-9v + 3)$
- d) $(-2g^2 - 12) - (-6g^2 + 4g - 1)$

11. Add or subtract. Use a strategy of your choice.

- a) $(3w^2 + 17w) + (12w^2 - 3w)$
- b) $(5m^2 - 3) + (m^2 + 3)$
- c) $(-3h - 12) - (-9h - 6)$
- d) $(6a^2 + 2a - 2) + (-7a^2 + 4a + 11)$
- e) $(3y^2 + 9y + 7) - (2y^2 - 4y + 13)$
- f) $(-14 + 3p^2 + 2p) - (-5p + 10 - 7p^2)$

12. a) Which polynomial must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?
 b) Which polynomial must be subtracted from $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$? Justify your answers.

**Start
Where You
Are**

How Can I Summarize What I Have Learned?

Suppose I want to summarize what I know about polynomials.

- What tools could I use to do this?
 - a Frayer model
 - a table
 - a concept map

I can use a Frayer model to explain the meaning of a term or concept.

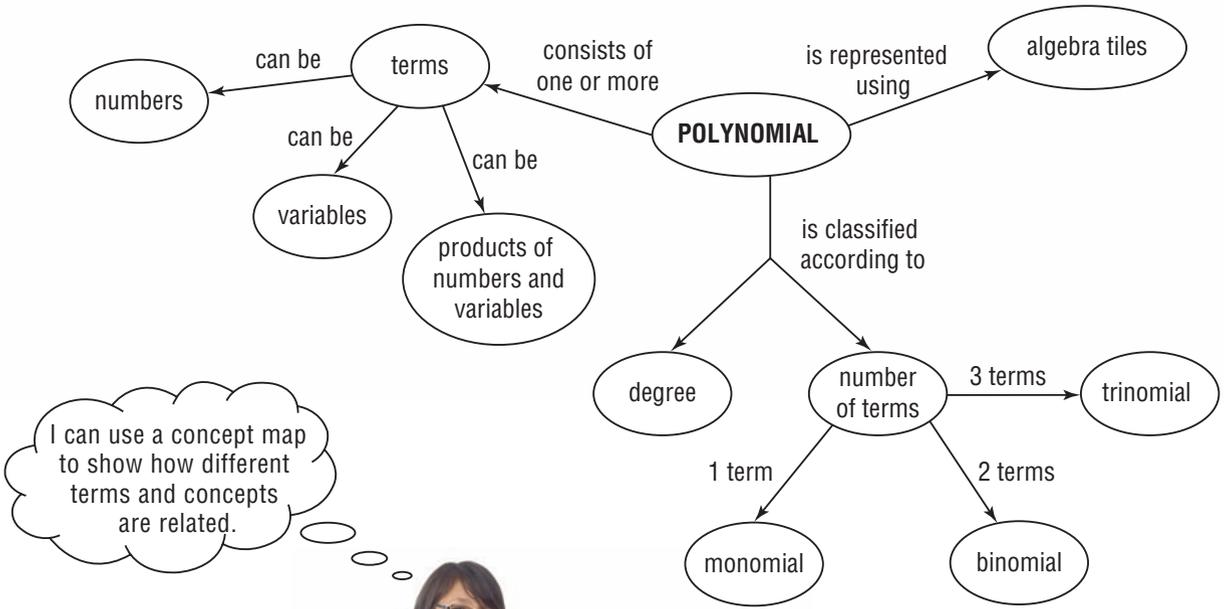


<p>Definition Like terms have the same variable raised to the same exponent.</p>	<p>Facts/Characteristics Like terms are represented by algebra tiles with the same size and shape. I can combine like terms by adding their coefficients.</p>
<p>Like terms</p>	
<p>Examples $-3x$ and $4x$ $5b^2$ and $2b^2$</p>	<p>Non-examples $-3c$ and 4 $5n^2$ and $2n$</p>

I can use a table to show how terms and concepts are alike and different.



Polynomial	Number of Terms	Name by Number of Terms	Degree
12	1	monomial	0
$8a$	1	monomial	1
$-4b^2 + 9$	2	binomial	2
$2c - 7$	2	binomial	1
$3d^2 - 4d + 6$	3	trinomial	2



I can use a concept map to show how different terms and concepts are related.



Check

Use the tools *you* find most helpful to summarize the important ideas and concepts you have learned about polynomials.

1. Choose another term or concept. Make a Frayer model to show what you know about that term or concept.
2. What other types of polynomials could you include in the table on page 238?
3. a) What could you add to the concept map above?
b) Think of another way to draw a concept map about polynomials.

Add to your Frayer model, table, or concept map as you work through this unit.



Investigating Polynomials that Generate Prime Numbers

A prime number is any whole number, greater than 1, that is divisible by only itself and 1.

In 1772, Leonhard Euler, a Swiss mathematician, determined that the polynomial $n^2 - n + 41$ generates prime numbers for different values of n .

Use a calculator to check that this is true:

- ▶ Choose a value of n between 1 and 10.
Substitute this number for n in the polynomial.
Is the number you get a prime number?
How do you know?
- ▶ Repeat the process for other values of n between 1 and 10.
- ▶ Choose a value of n between 10 and 40.
Substitute this number for n in the polynomial.
Is the number you get a prime number?
How do you know?
- ▶ Repeat the process for other values of n between 10 and 40.
- ▶ Substitute $n = 41$. Is the number you get a prime number?
How can you tell?
- ▶ List the values of n and the resulting primes in a table.

In 1879, E. B. Escott, an American mathematician, determined the polynomial $n^2 - 79n + 1601$ for generating prime numbers.

Test this polynomial:

- ▶ Substitute different values of n , and check that the numbers you get are prime. List the values of n and the resulting primes in a table. What patterns do you see?
- ▶ Substitute $n = 80$. Did you get a prime number? Explain.
- ▶ Determine other values of n for which Escott's polynomial does *not* generate prime numbers.

Currently, there is no known polynomial that generates only prime numbers. And, there is no known polynomial that generates all the prime numbers.

- ▶ Determine a value of n for which each of these polynomials does *not* generate a prime number:
 - $n^2 - n + 41$, other than $n = 41$
 - $n^2 - n + 17$
 - $n^2 + n - 1$

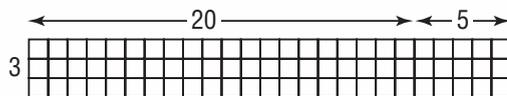
5.5

Multiplying and Dividing a Polynomial by a Constant

FOCUS

- Use different strategies to multiply and divide a polynomial by a constant.

How does this diagram model the product 3×25 ?



What property is illustrated by this diagram?

How could you use the diagram above to model division?

Investigate



Use any strategy or materials you wish.

► Determine each product. Write a multiplication sentence.

- $2(3x)$
- $3(2x + 1)$
- $2(2x^2 + x + 4)$
- $-2(3x)$
- $-3(2x + 1)$
- $-2(2x^2 + x + 4)$

► Determine each quotient. Write a division statement.

- $9x \div 3$
- $(8x + 12) \div 4$
- $(5x^2 + 10x + 20) \div 5$
- $9x \div (-3)$
- $(8x + 12) \div (-4)$
- $(5x^2 + 10x + 20) \div (-5)$

Reflect & Share

Compare your answers and strategies with those of another pair of students.

If your answers are different, find out why.

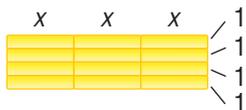
Look at your multiplication and division sentences.

What relationships do you see among the original terms and the answers?

How could you use these relationships to multiply and divide without using algebra tiles?

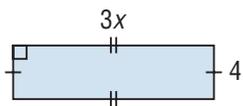
Connect

- The expression $4(3x)$ is a product statement. It represents the product of the constant, 4, and the monomial, $3x$. We can model the product as 4 rows of three x -tiles.



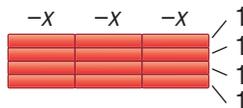
$$\begin{aligned} \text{So, } 4(3x) &= 3x + 3x + 3x + 3x && \text{This is repeated addition.} \\ &= 12x \end{aligned}$$

We can also model $4(3x)$ as the area of a rectangle with dimensions 4 and $3x$.



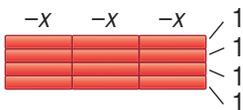
$$\begin{aligned} \text{So, } 4(3x) &= 4(3)(x) \\ &= 12x \end{aligned}$$

- $4(-3x)$ is the product of 4 and the monomial $-3x$. We can model the product as 4 rows of three $-x$ -tiles.



$$\begin{aligned} \text{So, } 4(-3x) &= -3x - 3x - 3x - 3x \\ &= -12x \end{aligned}$$

- $-4(3x)$ is the opposite of $4(3x)$. We can model this by flipping the tiles we used to model $4(3x)$.



$$\begin{aligned} \text{So, } -4(3x) &= -(12x) \\ &= -12x \end{aligned}$$

We can use the same strategy with algebra tiles to multiply a binomial or a trinomial by a constant. To determine the product symbolically, we use the *distributive property*.

Example 1 Multiplying a Binomial and a Trinomial by a Constant

Determine each product.

a) $3(-2m + 4)$

b) $-2(-n^2 + 2n - 1)$

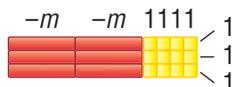
Solutions

Method 1

Use algebra tiles.

a) $3(-2m + 4)$

Display 3 rows of two $-m$ -tiles and four 1-tiles.

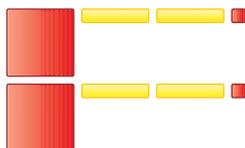


There are six $-m$ -tiles and twelve 1-tiles.

So, $3(-2m + 4) = -6m + 12$

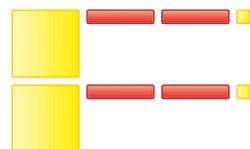
b) $-2(-n^2 + 2n - 1)$

Display 2 rows of one $-n^2$ -tile, two n -tiles, and one -1 -tile.



This shows $2(-n^2 + 2n - 1)$.

Flip all the tiles.



There are two n^2 -tiles, four $-n$ -tiles, and two 1-tiles.

So, $-2(-n^2 + 2n - 1) = 2n^2 - 4n + 2$

Method 2

Use the distributive property.

Multiply each term in the brackets by the term outside the brackets.

a) $3(-2m + 4) = 3(-2m) + 3(4)$
 $= -6m + 12$

b) $-2(-n^2 + 2n - 1)$
 $= (-2)(-n^2) + (-2)(2n) + (-2)(-1)$
 $= 2n^2 + (-4n) + 2$
 $= 2n^2 - 4n + 2$

Multiplication and division are inverse operations. To divide a polynomial by a constant, we reverse the process of multiplication.

- The expression $6x \div 3$ is a division statement.

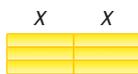
It represents the quotient of the monomial, $6x$, and the constant 3.

To model $6x \div 3$,

we arrange six x -tiles in 3 rows.

Each row contains two x -tiles.

So, $6x \div 3 = 2x$

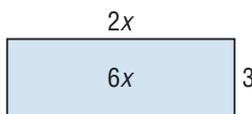


We can also model $6x \div 3$ as

one dimension of a rectangle with

an area of $6x$ and the other dimension 3.

Then, $6x \div 3 = \frac{6x}{3}$
 $= 2x$



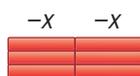
We can use what we know about division as a fraction and integer division to determine the quotient.

$$\begin{aligned}\frac{6x}{3} &= \frac{6}{3} \times x \\ &= 2 \times x \\ &= 2x\end{aligned}$$

- $(-6x) \div 3$ is the quotient of the monomial, $-6x$, and the constant 3.

Using a model:

We arrange six $-x$ -tiles in 3 rows.



Each row contains two $-x$ -tiles.

So, $(-6x) \div 3 = -2x$

Using fractions and integers:

$$(-6x) \div 3 = \frac{-6x}{3}$$

Simplify the fraction.

$$\begin{aligned}(-6x) \div 3 &= \frac{-6}{3} \times x \\ &= -2 \times x \\ &= -2x\end{aligned}$$

- $6x \div (-3)$ is the quotient of the monomial, $6x$, and the constant -3 .

Using fractions and integers:

$$6x \div (-3) = \frac{6x}{-3}$$

Simplify the fraction.

$$\begin{aligned}6x \div (-3) &= \frac{6}{-3} \times x \\ &= -2 \times x \\ &= -2x\end{aligned}$$

Example 2 Dividing a Binomial and a Trinomial by a Constant

Determine each quotient.

a) $\frac{4s^2 - 8}{4}$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

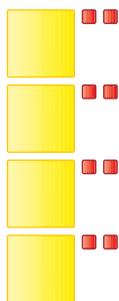
Solutions

Method 1

a) $\frac{4s^2 - 8}{4}$

Use algebra tiles.

Arrange four s^2 -tiles and eight -1 -tiles in 4 equal rows.



In each row, there is one s^2 -tile and two -1 -tiles.

So, $\frac{4s^2 - 8}{4} = s^2 - 2$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Think multiplication.

What do we multiply -3 by to get

$$-3m^2 + 15mn - 21n^2?$$

$$(-3) \times ? = -3m^2 + 15mn - 21n^2$$

$$\text{Since } (-3) \times 1 = -3,$$

$$\text{then } (-3) \times (1m^2) = -3m^2$$

$$\text{Since } (-3) \times (-5) = 15,$$

$$\text{then } (-3) \times (-5mn) = +15mn$$

$$\text{Since } (-3) \times 7 = -21,$$

$$\text{then } (-3) \times (+7n^2) = -21n^2$$

So, $\frac{-3m^2 + 15mn - 21n^2}{-3} = m^2 - 5mn + 7n^2$

Method 2

a) $\frac{4s^2 - 8}{4}$

Write the quotient expression as the sum of 2 fractions.

$$\frac{4s^2 - 8}{4} = \frac{4s^2}{4} + \frac{-8}{4}$$

Simplify each fraction.

$$= \frac{4}{4} \times s^2 + (-2)$$

$$= 1 \times s^2 - 2$$

$$= s^2 - 2$$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Write the quotient expression as the sum of 3 fractions.

$$\frac{-3m^2 + 15mn - 21n^2}{-3} = \frac{-3m^2}{-3} + \frac{15mn}{-3} + \frac{-21n^2}{-3}$$

Simplify each fraction.

$$= m^2 + (-5mn) + (7n^2)$$

$$= m^2 - 5mn + 7n^2$$

Discuss the ideas

- How could you use multiplication to verify the quotient in a division question?
- Why can we not use algebra tiles to divide when the divisor is negative?

Practice

Check

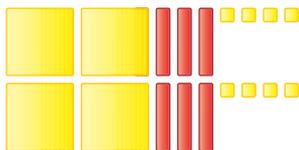
3. Write the multiplication sentence modelled by each set of algebra tiles.



4. For each set of algebra tiles in question 3, write a division sentence.

5. a) Which of these products is modelled by the algebra tiles below?

- $2(-2n^2 + 3n + 4)$
- $2(2n^2 - 3n + 4)$
- $-2(2n^2 - 3n + 4)$



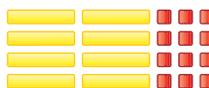
- b) In part a, two of the products were not modelled by the algebra tiles. Model each product. Sketch the tiles you used.

6. Which of these quotients is modelled by the algebra tiles below?

a) $\frac{8t - 12}{-4}$

b) $\frac{-8t - 12}{4}$

c) $\frac{8t - 12}{4}$



Apply

7. a) Multiply.

i) $3(5r)$

ii) $-3(5r)$

iii) $(5r)(3)$

iv) $-5(3r)$

v) $-5(-3r)$

vi) $(-3r)(5)$

- b) In part a, explain why some answers are the same.

- c) For which products in part a could you have used algebra tiles? For each product, sketch the tiles you could use.

8. a) Divide.

i) $\frac{12k}{4}$

ii) $(-12k) \div 4$

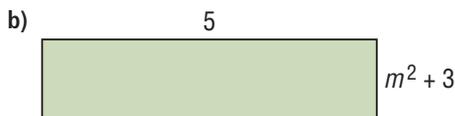
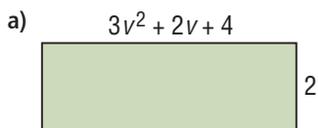
iii) $\frac{12k}{-4}$

iv) $(-12k) \div (-4)$

- b) In part a, explain why some answers are the same.

- c) For which quotients in part a could you have used algebra tiles? For each quotient, sketch the tiles you could use.

9. Write the multiplication sentence modelled by each rectangle.



10. For each rectangle in question 9, write a division sentence.

11. Use algebra tiles to determine each product. Sketch the tiles you used. Record the product symbolically.

- a) $7(3s + 1)$
 b) $-2(-7h + 4)$
 c) $2(-3p^2 - 2p + 1)$
 d) $-6(2v^2 - v + 5)$
 e) $(-w^2 + 3w - 5)(3)$
 f) $(x^2 + x)(-5)$

12. Here is a student's solution for this question:

Identify the errors in the solution, then write the correct solution.

13. Use algebra tiles to determine each quotient. Sketch the tiles you used. Record the product symbolically.

- a) $\frac{12p - 18}{6}$ b) $\frac{-6q^2 - 10}{2}$
 c) $\frac{5h^2 - 20h}{5}$ d) $\frac{4r^2 - 16r + 6}{2}$
 e) $\frac{-8a^2 + 4a - 12}{4}$ f) $\frac{6x^2 + 3x + 9}{3}$

14. Here is a student's solution for this question:
 Divide: $(-14m^2 - 28m + 7) \div (-7)$

Identify the errors in the solution, then write the correct solution.

15. Use any strategy to determine each product.

- a) $-3(-4u^2 + 16u + 8)$
 b) $12(2m^2 - 3m)$
 c) $(5t^2 + 2t)(-4)$
 d) $(-6s^2 - 5s - 7)(-5)$
 e) $4(-7y^2 + 3y - 9)$
 f) $10(8n^2 - n - 6)$

16. Use any strategy to determine each quotient.

- a) $\frac{24d^2 - 12}{12}$
 b) $\frac{8x + 4}{4}$
 c) $\frac{-10 + 4m^2}{-2}$
 d) $(25 - 5n) \div (-5)$
 e) $(-14k^2 + 28k - 49) \div 7$
 f) $\frac{30 - 36d^2 + 18d}{-6}$
 g) $\frac{-26c^2 + 39c - 13}{-13}$

17. Which pairs of expressions are equivalent?

Explain how you know.

- a) $5j^2 + 4$ and $5(j + 4)$
 b) $10x^2$ and $3x(x + 7)$
 c) $15x - 10$ and $5(-2 + 3x)$
 d) $-3(-4x - 1)$ and $12x^2 - 3x$
 e) $-5(3x^2 - 7x + 2)$ and $-15x^2 + 12x - 10$
 f) $2x(-3x - 7)$ and $-6x^2 - 14x$

18. Assessment Focus

- a) Determine each product or quotient.
- i) $(3p)(4)$ ii) $\frac{-21x}{3}$
- iii) $(3m^2 - 7)(-4)$
- iv) $\frac{-2f^2 + 14f - 8}{2}$
- v) $(6y^2 - 36y) \div (-6)$
- vi) $(-8n + 2 - 3n^2)(3)$
- b) List the products and quotients in part a that can be modelled with algebra tiles. Justify your selection.
- c) Sketch the tiles for one product and one quotient in part a.

19. a) Determine each product.

- i) $2(2x + 1)$ ii) $2(1 - 2x)$
- $3(2x + 1)$ $3(1 - 2x)$
- $4(2x + 1)$ $4(1 - 2x)$
- $5(2x + 1)$ $5(1 - 2x)$

- b) Describe the patterns in part a.
- c) Predict the next 3 products in each list in part a. How do you know the products are correct?
- d) Suppose you extended the lists in part a upward. Predict the preceding 3 products in each list.

20. a) The perimeter of an equilateral triangle is represented by the polynomial

$$15a^2 + 21a + 6.$$

Determine the polynomial that represents the length of one side.

- b) Determine the length of one side when $a = 4$ cm.

21. Square A has side length $4s + 1$. Square B has a side length that is 3 times as great as the side length of square A.

- a) What is the perimeter of each square? Justify your answers.
- b) Write a polynomial, in simplest form, to represent the difference in the perimeters of squares A and B.

22. Determine each product.

- a) $2(2x^2 - 3xy + 7y^2)$
- b) $-4(pq + 3p^2 + 3q^2)$
- c) $(-2gh + 6h^2 - 3g^2 - 9g)(3)$
- d) $5(-r^2 + 8rs - 3s^2 - 5s + 4r)$
- e) $-2(4t^2 - 3v^2 + 19tv - 6v - t)$

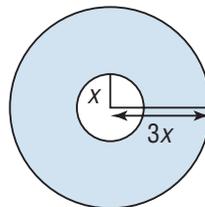
23. Determine each quotient.

- a) $(3n^2 - 12mn + 6m^2) \div 3$
- b) $\frac{-6rs - 16r - 4s}{-2}$
- c) $\frac{10gh - 30g^2 - 15h}{5}$
- d) $(12t^2 - 24ut - 48t) \div (-6)$

Take It Further

24. The area of a circle is given by the monomial πr^2 .

Write, then simplify a polynomial for the shaded area in this diagram:



Reflect

How are multiplying and dividing a polynomial by a constant related? Use examples to explain.

5.6

Multiplying and Dividing a Polynomial by a Monomial

FOCUS

- Use different strategies to multiply and divide a polynomial by a monomial.

You can use the strategies you know for multiplying and dividing a polynomial by a constant to multiply and divide a polynomial by a monomial.

Investigate

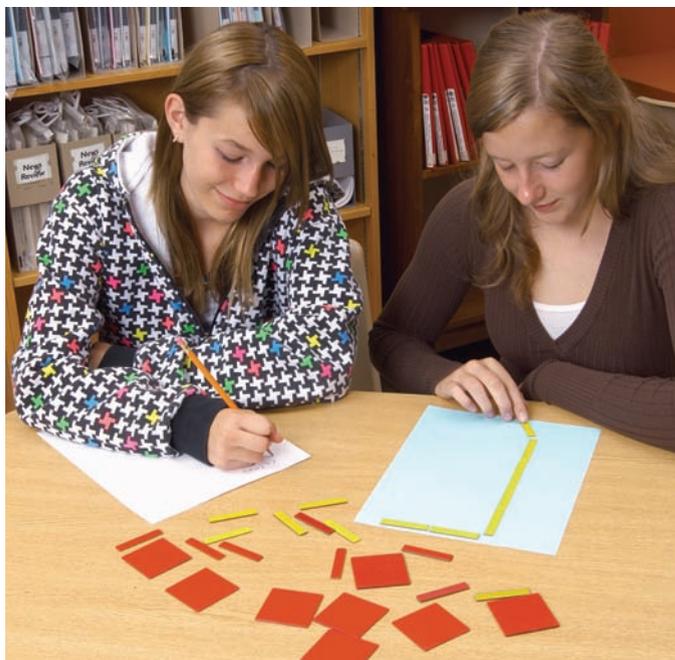
2

You may need algebra tiles.

- Determine each product.
Use a strategy of your choice.
Write a multiplication sentence.
 - $2a(5a)$
 - $4b(3b - 2)$
 - $-3c(-5c + 1)$

- Determine each quotient.
Use a strategy of your choice.
Write a division sentence.

$$\begin{aligned} & \frac{3g^2 + 9g}{3g} \\ & \frac{-18f^2 + 12f}{6f} \\ & \frac{24d^2 + 8d}{-4d} \end{aligned}$$



Reflect & Share

Compare your answers and strategies with those of another pair of students.

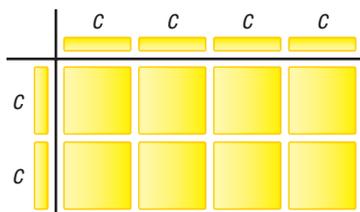
If you have different answers, find out why.

If you used different strategies, explain your strategies and choice of strategies.

How can you use multiplication to check your quotients?

Connect

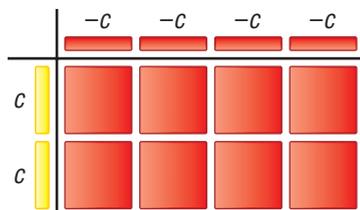
- The expression $(2c)(4c)$ is the product of two monomials. We interpret the product with algebra tiles arranged to form a rectangle with dimensions $2c$ and $4c$.



To help build the rectangle, we place guiding tiles to represent each dimension. Then we fill in the rectangle with tiles.

We need eight c^2 -tiles to build the rectangle.
So, $(2c)(4c) = 8c^2$

- The expression $(2c)(-4c)$ is the product of a positive and a negative monomial. We form a rectangle with guiding tiles: two c -tiles along one dimension and four $-c$ -tiles along the other dimension. We know that the product of a positive number and a negative number is negative. So, when we fill in the rectangle, we use $-c^2$ -tiles.



We need eight $-c^2$ -tiles to build this rectangle.
So, $(2c)(-4c) = -8c^2$

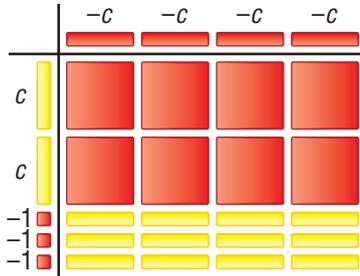
We use similar strategies to multiply a binomial by a monomial.

- The expression $-4c(2c - 3)$ is the product of a monomial and a binomial. We form a rectangle with guiding tiles:
- four $-c$ -tiles along one dimension; and
 - two c -tiles and three -1 -tiles along the other dimension

The product of two numbers with opposite signs is negative. So, when we place a tile in a row and column headed by guiding tiles with opposite signs, the tile is negative.

The product of two numbers with the same sign is positive.

So, when we place a tile in a row and column headed by guiding tiles with the same sign, the tile is positive.



There are eight $-c^2$ -tiles and twelve c -tiles.

$$\text{So, } -4c(2c - 3) = -8c^2 + 12c$$

Example 1 Multiplying a Binomial by a Monomial

Determine each product.

a) $2x(3x + 4)$

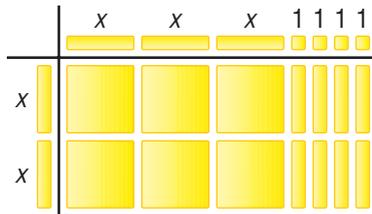
b) $-2x(-3x + 4)$

Solutions

Method 1

a) $2x(3x + 4)$

Use algebra tiles to make a rectangle with dimensions $2x$ and $3x + 4$.



Six x^2 -tiles and eight x -tiles fill the rectangle.

$$\text{So, } 2x(3x + 4) = 6x^2 + 8x$$

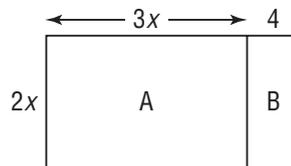
Method 2

a) $2x(3x + 4)$

Use an area model.

Sketch a rectangle with dimensions $2x$ and $3x + 4$.

Divide the rectangle into 2 smaller rectangles.



Rectangle A has area: $2x(3x) = 6x^2$

Rectangle B has area: $2x(4) = 8x$

The total area is: $6x^2 + 8x$

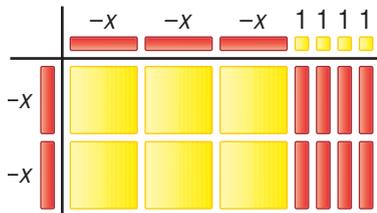
$$\text{So, } 2x(3x + 4) = 6x^2 + 8x$$

b) $-2x(-3x + 4)$

Use algebra tiles.

Form a rectangle with guiding tiles:

- two $-x$ -tiles along one dimension; and
- three $-x$ -tiles and four 1-tiles along the other dimension



Six x^2 -tiles and eight $-x$ -tiles fill the rectangle.

So, $-2x(-3x + 4) = 6x^2 - 8x$

b) $-2x(-3x + 4)$

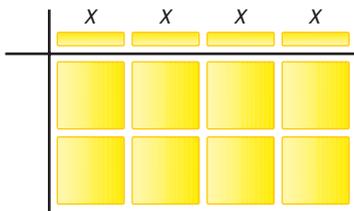
Use the distributive property.

Multiply each term in the brackets by the term outside the brackets.

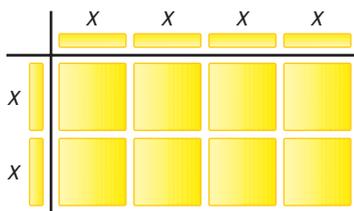
$$\begin{aligned} -2x(-3x + 4) &= -2x(-3x) + (-2x)(4) \\ &= 6x^2 - 8x \end{aligned}$$

To divide a polynomial by a monomial, we reverse the process of multiplying these polynomials.

- To determine the quotient of $\frac{8x^2}{4x}$, arrange eight x^2 -tiles in a rectangle with one dimension $4x$.



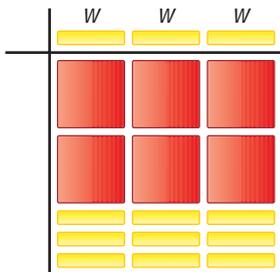
Along the left side of the rectangle, the guiding tiles are x -tiles.



There are 2 guiding x -tiles.

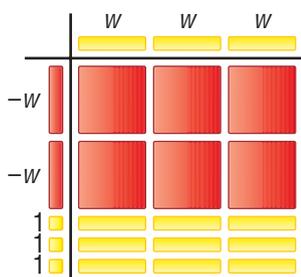
So, $\frac{8x^2}{4x} = 2x$

- To determine the quotient of $\frac{-6w^2 + 9w}{3w}$, arrange six $-w^2$ -tiles and nine w -tiles in a rectangle with one dimension $3w$.



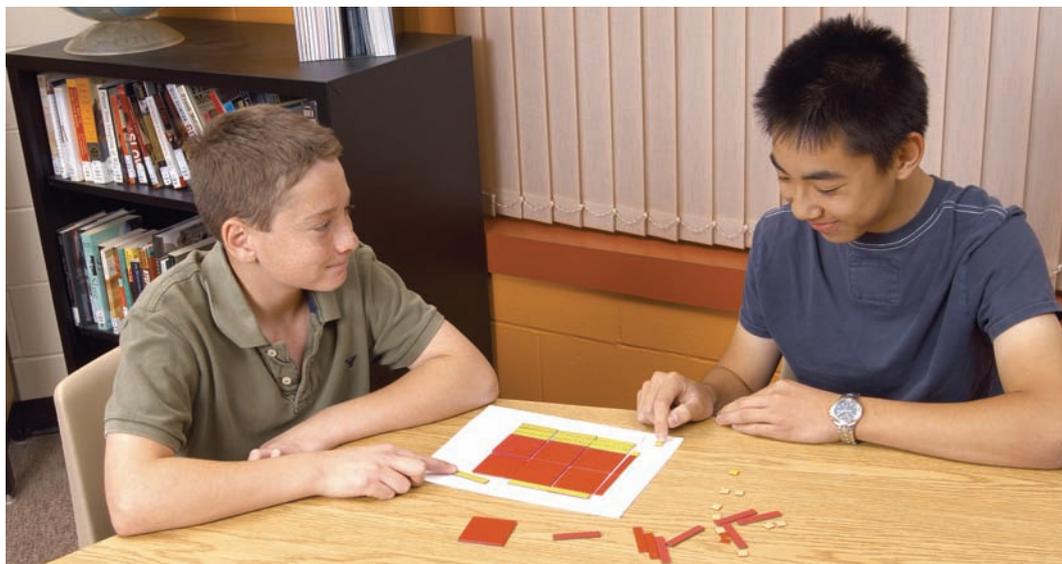
Along the left side of the rectangle:

- the guiding w -tiles are negative because they must have the sign opposite to that of the guiding tiles along the top of the rectangle
- the guiding 1-tiles are positive because they must have the same sign as the guiding tiles along the top of the rectangle



There are 2 guiding $-w$ -tiles and 3 guiding 1-tiles.

$$\text{So, } \frac{-6w^2 + 9w}{3w} = -2w + 3$$



Example 2

Dividing a Monomial and a Binomial by a Monomial

Determine each quotient.

a) $\frac{-10m^2}{2m}$

b) $\frac{30k^2 - 18k}{-6k}$

Solutions

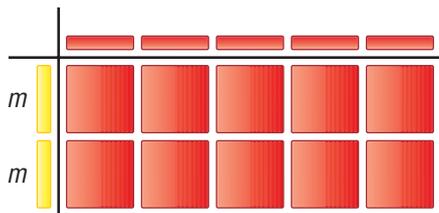
Method 1

a) $\frac{-10m^2}{2m}$

Use algebra tiles.

Arrange ten $-m^2$ -tiles in a rectangle with one dimension $2m$.

The guiding tiles along the other dimension represent $-5m$.



So, $\frac{-10m^2}{2m} = -5m$

b) $\frac{30k^2 - 18k}{-6k}$

Think multiplication.

$$-6k \times ? = 30k^2 - 18k$$

$$\text{Since } -6k \times (-5k) = 30k^2 \text{ and}$$

$$-6k \times (+3) = -18k$$

$$\text{Then } -6k \times (-5k + 3) = 30k^2 - 18k$$

So, $\frac{30k^2 - 18k}{-6k} = -5k + 3$

Method 2

a) $\frac{-10m^2}{2m}$

Think multiplication.

$$2m \times ? = -10m^2$$

$$\text{Since } 2 \times (-5) = -10 \text{ and } m \times m = m^2$$

$$\text{Then } 2m \times (-5m) = -10m^2$$

So, $\frac{-10m^2}{2m} = -5m$

b) $\frac{30k^2 - 18k}{-6k}$

Write the quotient expression as the sum of two fractions.

$$\frac{30k^2 - 18k}{-6k} = \frac{30k^2}{-6k} + \frac{-18k}{-6k}$$

Simplify each fraction.

$$\begin{aligned} \frac{30k^2 - 18k}{-6k} &= \frac{30}{-6} \times \frac{k^2}{k} + \frac{-18}{-6} \times \frac{k}{k} \\ &= (-5) \times k + 3 \times 1 \\ &= -5k + 3 \end{aligned}$$

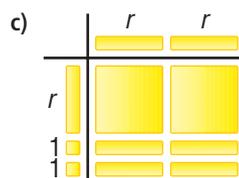
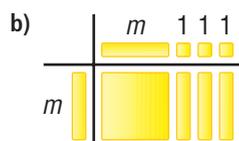
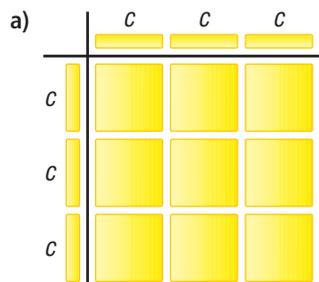
Discuss the ideas

1. Why can we not use repeated addition to model the product $(2c)(4c)$?
2. Why can we not use an area model to multiply when there are negative terms in the product statement?
3. How could we check that a quotient is correct?

Practice

Check

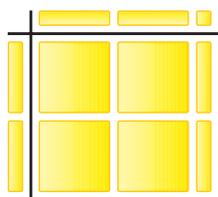
4. Write the multiplication sentence modelled by each set of algebra tiles.



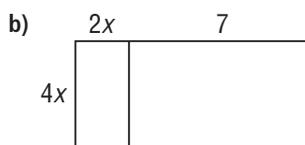
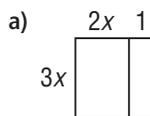
5. For each set of algebra tiles in question 4, write a division sentence.

6. Which of these multiplication sentences is modelled by the algebra tiles below?

- a) $2n(n + 2)$
 b) $2(2n^2 + 1)$
 c) $2n(2n + 1)$



7. Write the multiplication sentence modelled by each rectangle.



8. For each rectangle in question 7, write a division sentence.

Apply

9. a) Multiply.

- i) $(3m)(4m)$ ii) $(-3m)(4m)$
 iii) $(3m)(-4m)$ iv) $(-3m)(-4m)$
 v) $(4m)(3m)$ vi) $(4m)(-3m)$

- b) In part a, explain why there are only two answers.
 c) For which products in part a could you have used algebra tiles? For each product, sketch the tiles you could use.

10. a) Divide.

- i) $\frac{12x}{2x}$ ii) $\frac{12x}{-2x}$
 iii) $\frac{-12x}{2x}$ iv) $\frac{-12x}{-2x}$
 v) $\frac{12x^2}{2x}$ vi) $\frac{12x^2}{2x^2}$
 vii) $\frac{-12x^2}{2x^2}$ viii) $\frac{12x^2}{-2x^2}$

- b) In part a, explain why some quotients are equal.
 c) For which quotients in part a could you have used algebra tiles? For each quotient, sketch the tiles you could use.

11. Multiply or divide as indicated.

- a) $(2r)(-6r)$
- b) $(-16n^2) \div (-8n)$
- c) $(-5g)(7g)$
- d) $\frac{40k}{-10k}$
- e) $(9h)(3h)$
- f) $\frac{48p^2}{12p}$
- g) $18u^2 \div (-3u^2)$
- h) $\frac{-24d^2}{-8d^2}$

12. Use any strategy to determine each product.

- a) $2x(x + 6)$
- b) $3t(5t + 2)$
- c) $-2w(3w - 5)$
- d) $-x(2 + 8x)$
- e) $3g(-5 - g)$
- f) $(4 + 3y)(2y)$
- g) $(-7s - 1)(-y)$
- h) $(-3 + 6r)(2r)$

13. A student thinks that the product $2x(x + 1)$ is $2x^2 + 1$. Choose a model. Use the model to explain how to get the correct answer.

14. Here is a student's solution for this question:
Multiply: $(-2d + 9)(-3d)$

$$\begin{aligned} &(-2d + 9)(-3d) \\ &= (-2d)(-3d) - (9)(-3d) \\ &= -6d^2 - (27d) \\ &= -6d^2 - 27d \end{aligned}$$

Identify the errors in the solution, then write the correct solution.

15. a) Describe two different strategies to simplify $\frac{3r^2 - 12r}{3r}$.
b) Which strategy do you find easier? Explain.

16. Use any strategy to determine each quotient.

- a) $\frac{10x^2 + 4x}{2x}$
- b) $(6x^2 + 4x) \div x$
- c) $\frac{6y + 3y^2}{3y}$
- d) $\frac{40x^2 - 16x}{8x}$
- e) $\frac{15g - 10g^2}{5g}$
- f) $\frac{-12k - 24k^2}{3k}$
- g) $(24h^2 + 36h) \div (-4h)$
- h) $(-8m^2 + 18m) \div (-2m)$

17. Assessment Focus

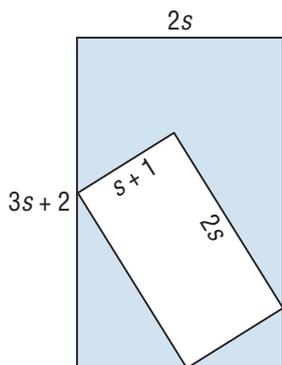
- a) Determine each product or quotient. Use a different strategy each time.
 - i) $\frac{15n^2 + 5n}{5n}$
 - ii) $-3r(4 - 7r)$
 - iii) $(-16s^2 + 4s) \div (-2s)$
 - iv) $(t - 9)(4t)$

b) Choose one product and one quotient in part a. Use a different strategy to solve each problem. In each case, which strategy do you prefer? Explain why.

18. a) Use algebra tiles to model the quotient $\frac{12x^2 + 12x}{2x}$. Determine the quotient.

b) The polynomial $12x^2 + 12x$ can be represented by the areas of rectangles with different dimensions. Sketch and label the dimensions for as many different rectangles as you can. For each rectangle, write a division statement.

19. a) Write a polynomial to represent the area of each rectangle in the diagram below.



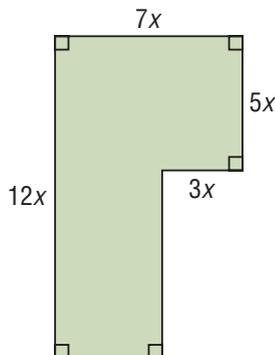
- b) Determine a polynomial for the shaded area. Justify your strategy.
 c) Determine the area in part b when $s = 2.5$ cm.
20. Determine each product.
- $3m(2n + 4)$
 - $(-5 + 3f)(-2g)$
 - $7m(-6p + 7m)$
 - $(-8h - 3k)(4k)$
 - $(-2t + 3r)(4t)$
 - $(-g)(8h - 5g)$

21. Determine each quotient.

- $(12x^2 + 6xy) \div 3x$
- $\frac{12gh + 6g}{2g}$
- $(-27p^2 + 36pq) \div 9p$
- $\frac{40rs - 35r}{-5r}$
- $\frac{14n^2 + 42np}{-7n}$

Take It Further

22. Determine a polynomial for the area of this shape. Justify your answer.



23. a) The polynomial $54s^2$ represents the surface area of a cube. Determine a polynomial that represents the area of one face.
 b) Use your answer to part a. Determine the length of an edge of the cube.
24. The product $2\pi r(r + h)$ represents the surface area of a cylinder.
- Determine the product.
 - To check your work, determine the surface area of a cylinder with radius 5 cm and height 3 cm two ways:
 - using the product
 - using your answer to part a
25. Simplify:
 $[(2x^2 - 8x + 3xy + 5) + (24x^2 - 16x - 12xy)] \div 4x$

Reflect

Explain how the strategies for dividing a polynomial by a monomial are related to the strategies for multiplying a polynomial by a monomial. Include examples in your explanation.

Study Guide

Polynomials

- ▶ A polynomial is one term or the sum of terms whose variables have whole-number exponents; for example, $2m^2 + 3m - 5$
- ▶ The numerical value of a term is its coefficient.
- ▶ A term that consists of only a number is a constant term.
- ▶ The degree of a polynomial in the variable m is the highest power of m in the polynomial.
- ▶ A polynomial with: 1 term is a monomial; 2 terms is a binomial; and 3 terms is a trinomial.

Algebra Tiles

We can represent a polynomial with algebra tiles.

$$2p^2 + 2p - 3$$



Like Terms

Like terms are represented by the same type of algebra tile. In symbolic form, like terms have the same variables raised to the same exponent. Like terms can be added or subtracted. $3x^2$ and $2x^2$ are like terms, but $-x$ and 3 are not.

$$3x^2:$$



$$2x^2:$$



$$3x^2 + 2x^2 \text{ simplifies to } 5x^2.$$

$$-x:$$



$$3:$$



$$-x + 3 \text{ cannot be simplified.}$$

Operations with Polynomials

We can use algebra tiles to model operations with polynomials, then record the answers symbolically.

- ▶ To add polynomials, combine like terms:

$$\begin{aligned}(3r^2 + 5r) + (2r^2 - r) &= 3r^2 + 5r + 2r^2 - r \\ &= 5r^2 + 4r\end{aligned}$$

- ▶ To subtract polynomials, use a strategy for subtracting integers:

$$\begin{aligned}(3r^2 + 5r) - (2r^2 - r) &= 3r^2 + 5r - (2r^2) - (-r) \\ &= 3r^2 - 2r^2 + 5r + r \\ &= r^2 + 6r\end{aligned}$$

- ▶ To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial: $2t(5t - 3) = 2t(5t) + 2t(-3)$

$$= 10t^2 - 6t$$

- ▶ To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$\begin{aligned}\frac{21x^2 - 14x}{7x} &= \frac{21x^2}{7x} - \frac{14x}{7x} \\ &= 3x - 2\end{aligned}$$

Review

5.1

1. Use algebra tiles to model each polynomial. Sketch the tiles you used.

a) $2u^2 + 5u$ b) $4n^2 - 2n - 3$

2. Identify the variables, coefficients, and constant terms in each polynomial.

a) $4w - 3$ b) $5v^2 + 3$ c) $5y - 6 - y^2$

3. Classify each polynomial below:

- i) according to the number of terms
- ii) according to its degree

a) $3f + 5$ b) $-2g^2$ c) $5h - 6 - h^2$

4. Use algebra tiles to model the polynomial that fits each description. Sketch the tiles you used.

- a) a second-degree trinomial in the variable y , the coefficients of the variable when the polynomial is written in descending order are -1 and -3 , and with constant term 4
- b) a first-degree binomial in the variable x , with constant term 4 , and the coefficient of the other term is -3

5. Identify the equivalent polynomials.

Explain how you know they are equivalent.

- a) $-3x^2 + 3x - 11$ b) $3x^2 + 4x$
- c) $-2 - x$ d) $7 + 5x$
- e) $5x + 7$ f) $x - 2$
- g) $4x + 3x^2$ h) $3x - 11 - 3x^2$

6. Which polynomial is modelled by each set of algebra tiles?

State the degree of the polynomial.



7. Jennie does not understand how the terms $2k$ and k^2 are different. Use algebra tiles to model these terms and explain the difference.

8. For each polynomial, write an equivalent polynomial.

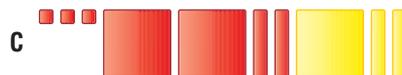
a) $-1 - 2h$ b) $3j + 2j^2 - 4$ c) $-5p + p^2$

9. Identify like terms.

- a) $5x^2, 3y^2, -2x^2, 5x, 2y$
- b) $-8x, 5x, 8, -2, -x, 11$

10. Match each algebra tile model below with its corresponding polynomial.

- a) $n^2 - n + 3$ b) $-w^2 - 3$
- c) $-2t$ d) $2q + 2$
- e) $2r^2 - 2r + 1$



11. Write an expression with 5 terms that has only 3 terms when simplified.

5.3
5.4

12. Simplify by combining like terms.

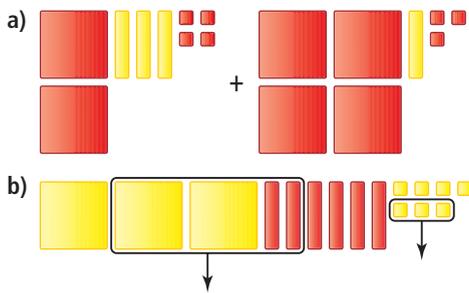
- $3x + 4 - 2x - 8 + 3x - 3$
- $4y^2 - 2y + 3y - 11y^2$
- $2a^2 + 7a - 3 - 2a^2 - 4a + 6$
- $2a^2 + 3a + 3a^2 - a^2 - a - 4a^2$

13. Students who have trouble with algebra often make these mistakes.

- ▶ They think: $x + x = x^2$
- ▶ They think: $(x)(x) = 2x$

Use algebra tiles to explain the correct answers.

14. Write the polynomial sum or difference modelled by each set of tiles. Determine the sum or difference.



15. Add or subtract as indicated.

- $(p^2 + 3p + 5) + (3p^2 + p + 1)$
- $(3q^2 + 3q + 7) - (2q^2 + q + 2)$
- $(6 - 3r + 7r^2) - (9 + 4r + 3r^2)$
- $(5s + 3 - s^2) + (5 + 3s - 2s^2)$
- $(-4t^2 - 3t + 9) - (-2t^2 - 5t - 1)$
- $(-9u^2 - 5) - (-3u^2 - 9)$
- $(3a^2 + 5ab - 7b^2) + (3b^2 - 10ab - 7a^2)$
- $(10xy - 3y^2 + 2x) - (5y - 4x^2 + xy)$

5.5

16. The sum of two polynomials is $15c + 6$. One polynomial is $3c - 7$. What is the other polynomial? Explain how you found it.

17. Match each sum or difference of polynomials with its answer. Justify your choices.

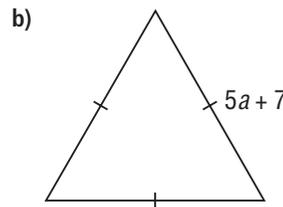
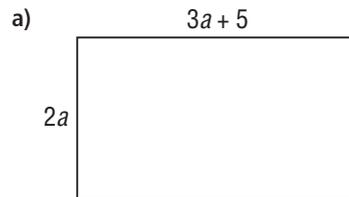
A	$(5x^2 - 2) + (2x^2 + 4)$	P	$4x^2 + 2x - 1$
B	$(x^2 - 3x) - (4x^2 - x)$	Q	$7x^2 + 2$
C	$(x^2 + 2x + 3) + (3x^2 - 4)$	R	$x^2 + 2x - 1$
D	$(3x^2 - x + 2) - (2x^2 - 3x + 3)$	S	$-3x^2 - 2x$
E	$(-3x - 2) - (3x - 2)$	T	$-6x$

18. The difference of two polynomials is $3d^2 - 7d + 4$.

One polynomial is $-8d^2 - 5d + 1$.

- What is the other polynomial? Explain how you found it.
- How many different answers can you find?

19. Write a polynomial for the perimeter of each shape. Simplify the polynomial. Determine each perimeter when $a = 3$ cm.



20. Write the multiplication sentence modelled by each set of algebra tiles.



21. For each set of algebra tiles in question 20, write a division sentence.

22. Determine each product or quotient.

Use any strategy you wish.

a) $10k \div 2$ b) $5(-4x^2)$

c) $2(-3m + 4)$ d) $\frac{-6n^2}{3}$

e) $-3(4s - 1)$ f) $\frac{9 - 12m}{3}$

g) $5(-7 + 2x)$

h) $-2(1 - 2n + 3n^2)$

i) $2(x + 3x^2)$

j) $(-6p^2 - 6p + 4) \div (-2)$

k) $\frac{15 - 21q + 6q^2}{-3}$

l) $(2 + 5n - 7n^2)(-6)$

23. Determine each product or quotient.

a) $(xy - x^2 + y^2)(-2)$

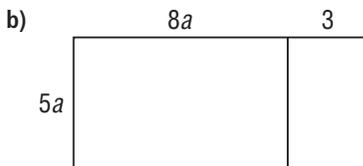
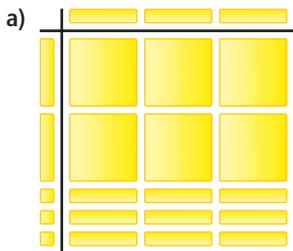
b) $(12m^2 - 6n + 8m) \div (-2)$

c) $\frac{-18pq + 3p^2 - 9q}{3}$

d) $4(2r^2 - 3r + 4s - 5s^2)$

5.6

24. Write the multiplication sentence modelled by each diagram.



25. Write a division sentence for each diagram in question 24.

26. Determine each product.

a) $(7s)(2s)$

b) $(-3g)(-5g)$

c) $m(3m + 2)$

d) $-5t(t - 3)$

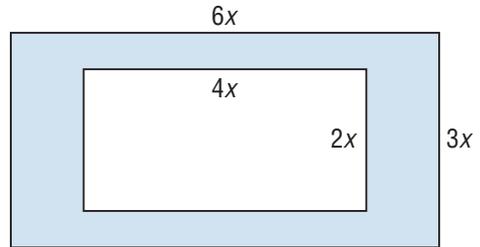
e) $7z(-4z - 1)$

f) $(-3f - 5)(-2f)$

g) $-5k(3 - k)$

h) $y(1 - y)$

27. This diagram shows one rectangle inside another.



a) Determine the area of each rectangle.

b) Determine the area of the shaded region. Explain your strategy.

28. Determine each quotient.

a) $24j \div (-6j)$

b) $\frac{24x}{3x}$

c) $\frac{-36x^2}{-9x}$

d) $(-8a^2 - 12a) \div 4a$

e) $(-8c + 4c^2) \div 4c$

f) $\frac{14y^2 - 21y}{-7y}$

29. a) The area of a rectangular deck is $(8d^2 + 20d)$ square metres. The deck is $4d$ metres long. Determine a polynomial that represents the width of the deck.
b) What are the dimensions and area of the deck when d is 4 metres?

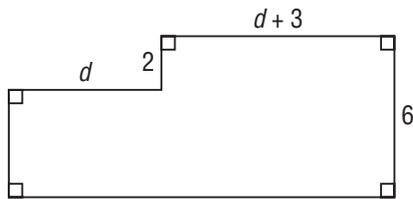


Practice Test

1. a) Which polynomial in t do these tiles represent?



- b) Classify the polynomial by degree and by the number of terms.
 c) Identify the constant term and the coefficient of the t^2 -term.
2. a) Write a polynomial for the perimeter of this shape. Simplify the polynomial.



- b) Determine the perimeter of the shape when $d = 5$ m.
3. Sketch algebra tiles to explain why:
 a) $3x + 2x$ equals $5x$ b) $(3x)(2x)$ equals $6x^2$
4. A student determined the product $3r(r + 4)$.
 The student's answer was $3r^2 + 4$.
 Use a model to explain whether the student's answer is correct.
5. Add or subtract as indicated. What strategy will you use each time?
 a) $(15 - 3d) + (3 - 15d)$ b) $(9h + 3) - (9 - 3h^2)$
 c) $(2y^2 + 5y - 6) + (-7y^2 + 2y - 6)$ d) $(7y^2 + y) - (3y - y^2)$
6. Multiply or divide as indicated. What strategy will you use each time?
 a) $25m(3m - 2)$ b) $-5(3v^2 - 2v - 1)$
 c) $(8x^2 - 4x) \div 2x$ d) $\frac{-6 + 3g^2 - 15g}{-3}$
7. Determine two polynomials with:
 a) a sum of $3x^2 - 4x - 2$
 b) a difference of $3x^2 - 4x - 2$
8. A rectangle has dimensions $5s$ and $3s + 8$.
 a) Sketch the rectangle and label it with its dimensions.
 b) What is the area of the rectangle?
 c) What is the perimeter of the rectangle?

Unit Problem

Algebra Patterns on a 100-Chart

You will need a copy of a 100-chart.

- Choose any 3 by 3 square of numbers on the chart. Add the numbers in each diagonal. What do you notice?
- Choose a different 3 by 3 square. Add the numbers in each diagonal. How do your results compare?
- Determine a relationship between the number at the centre of any 3 by 3 square and the sum of the numbers in a diagonal.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Let x represent the number at the centre of any 3 by 3 square. Write a polynomial, in terms of x , for each number at the four corners of the square.
- Add the polynomials in each diagonal. What is the sum? How does this explain the relationship you found earlier?
- Suppose you know the sum of the numbers in a diagonal of a 3 by 3 square. How could you determine the number at the centre of the square?
- What do you think is the relationship between the number at the centre of a 5 by 5 square and the sum of the numbers in a diagonal? What about a 7 by 7 square? Make a prediction, then use polynomials to check.

?		?
	x	
?		?

Your work should show:

- each 3 by 3 square and the related calculations
- a relationship between the number at the centre and the sum
- how this relationship changes as the size of the square changes

Reflect

on Your Learning

What did you find easy about polynomials? What did you find difficult? What strategies might you use to overcome these difficulties?